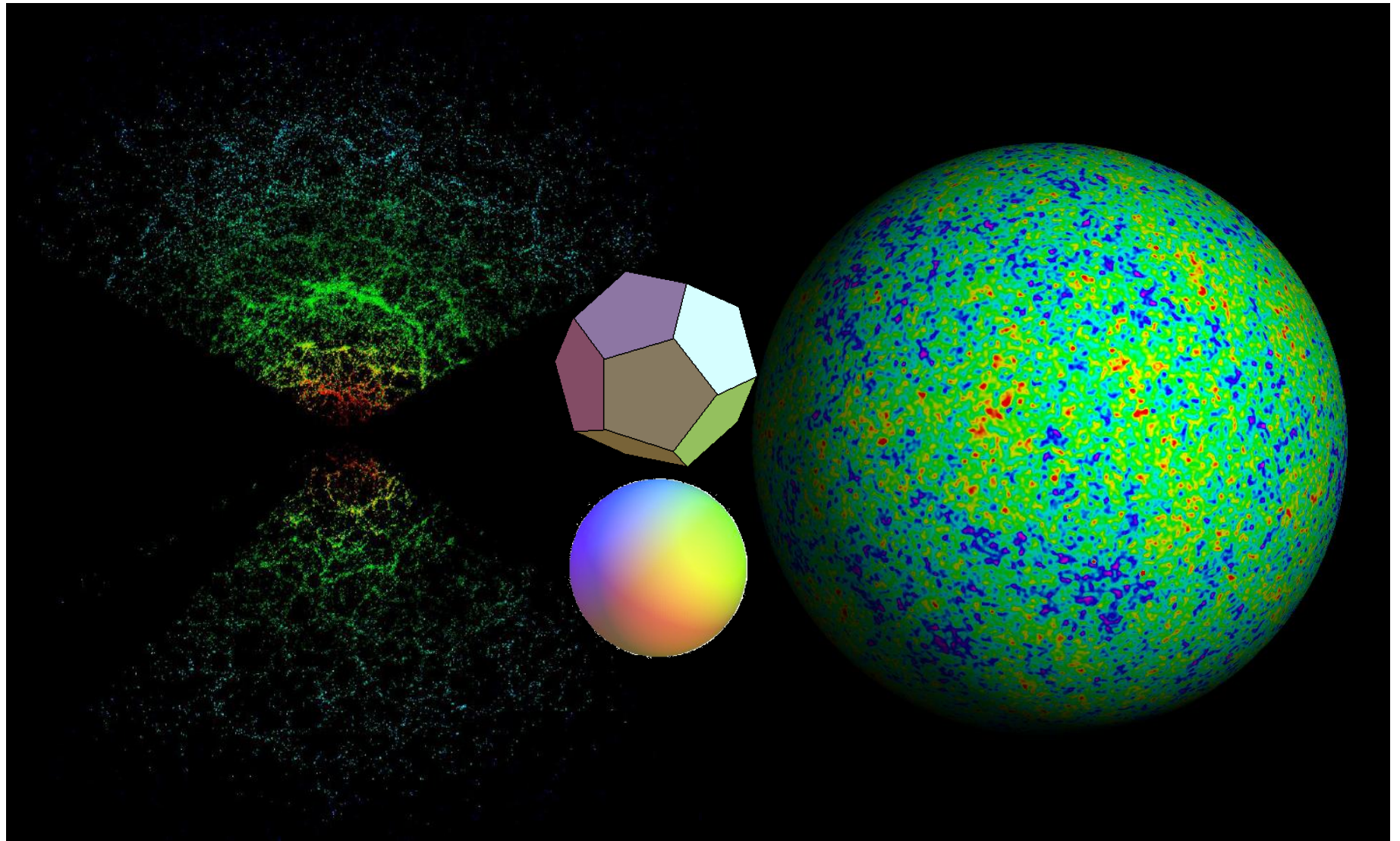
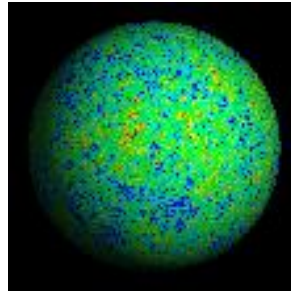


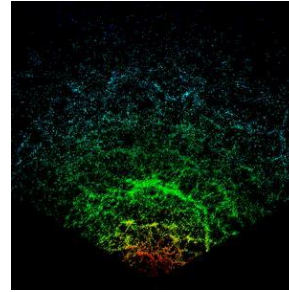
The Mathematical Universe



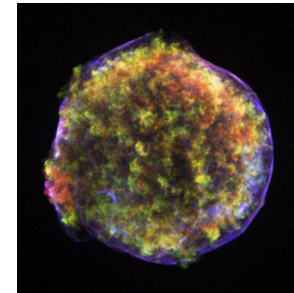
Max Tegmark, MIT



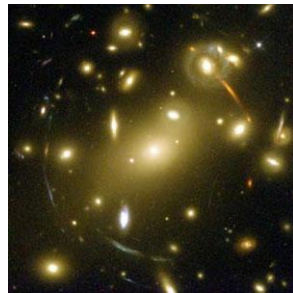
Microwave background



Galaxy surveys

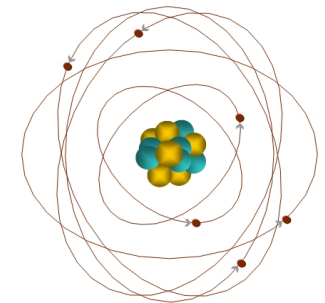


Supernovae Ia

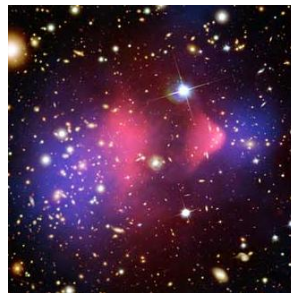


Gravitational lensing

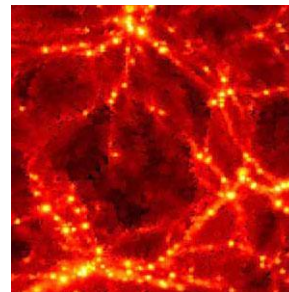
THE COSMIC SMÖRGÅSBORD



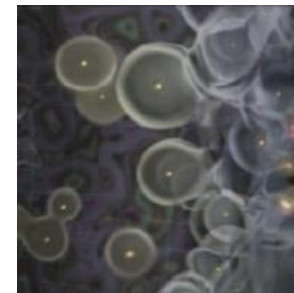
Big Bang nucleosynthesis



Galaxy clusters



Lyman α forest



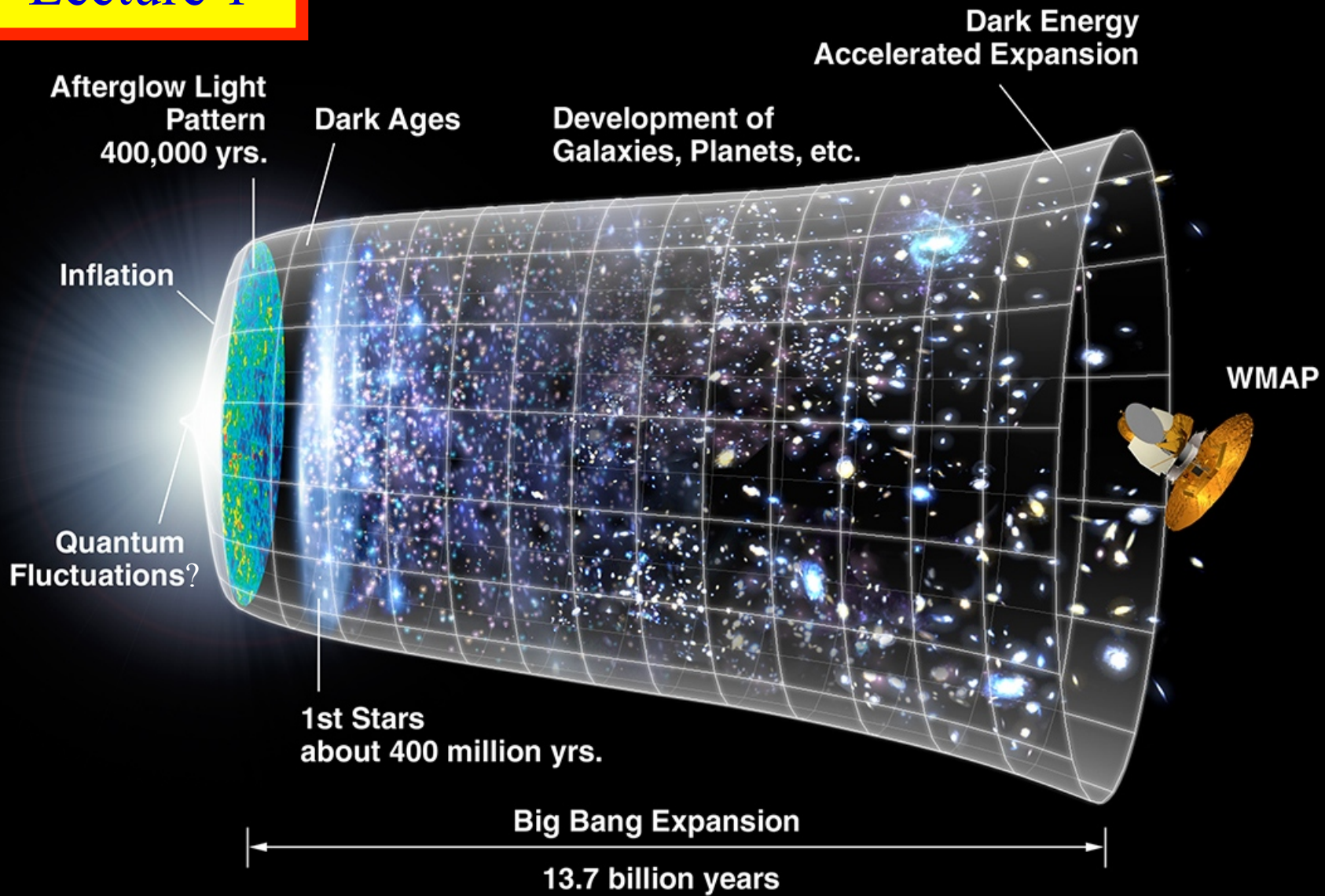
Neutral hydrogen tomography



Max Tegmark
Dept. of Physics, MIT
tegmark@mit.edu
Bright Horizons Cruise
May 13, 2011

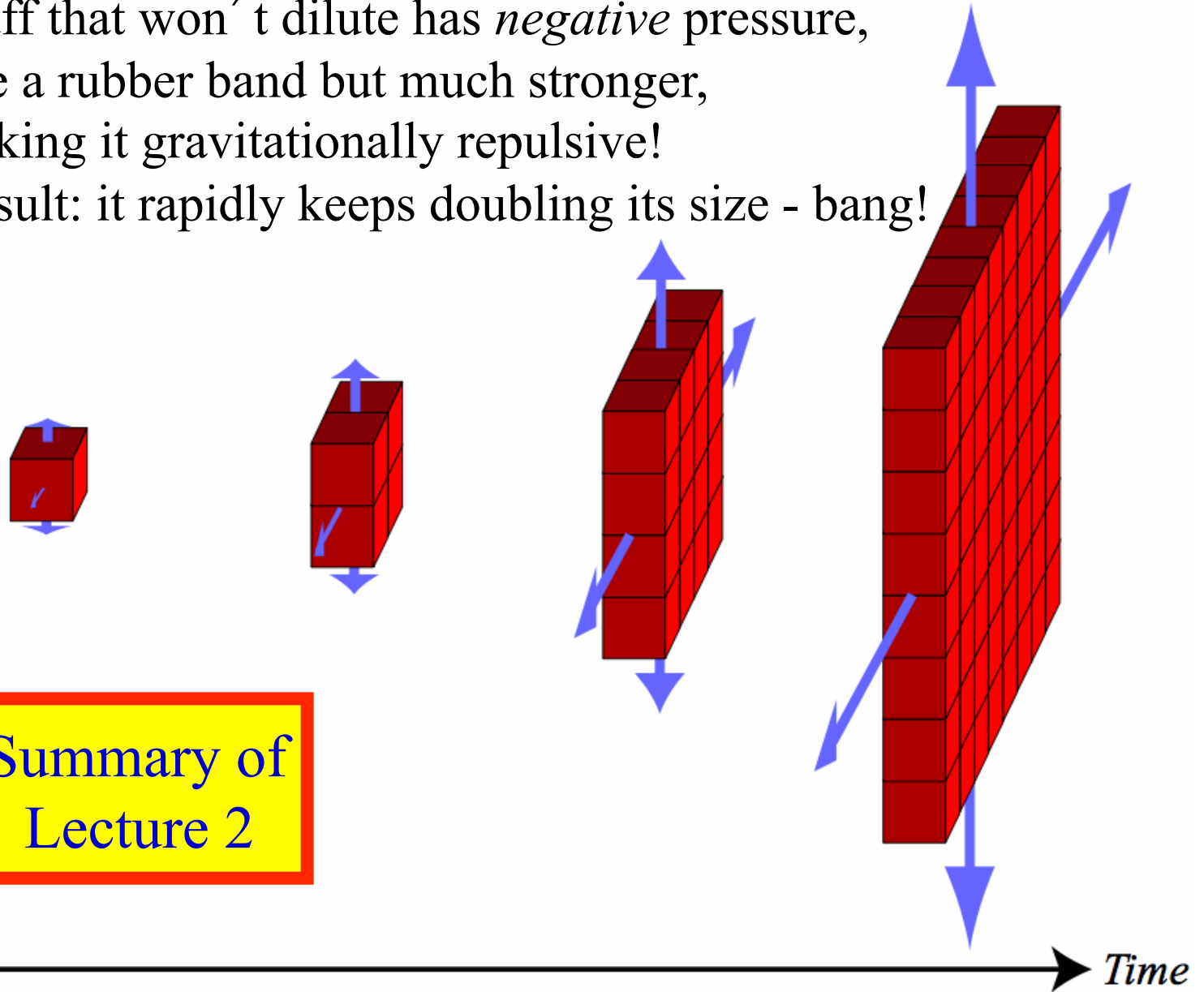
What have
we learned?

Summary of Lecture 1

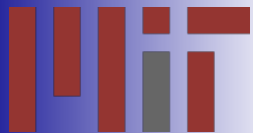


How inflation works:

- Einstein: source of gravity = density $+3p/c^2$
- Stuff that won't dilute has *negative* pressure, like a rubber band but much stronger, making it gravitationally repulsive!
- Result: it rapidly keeps doubling its size - bang!

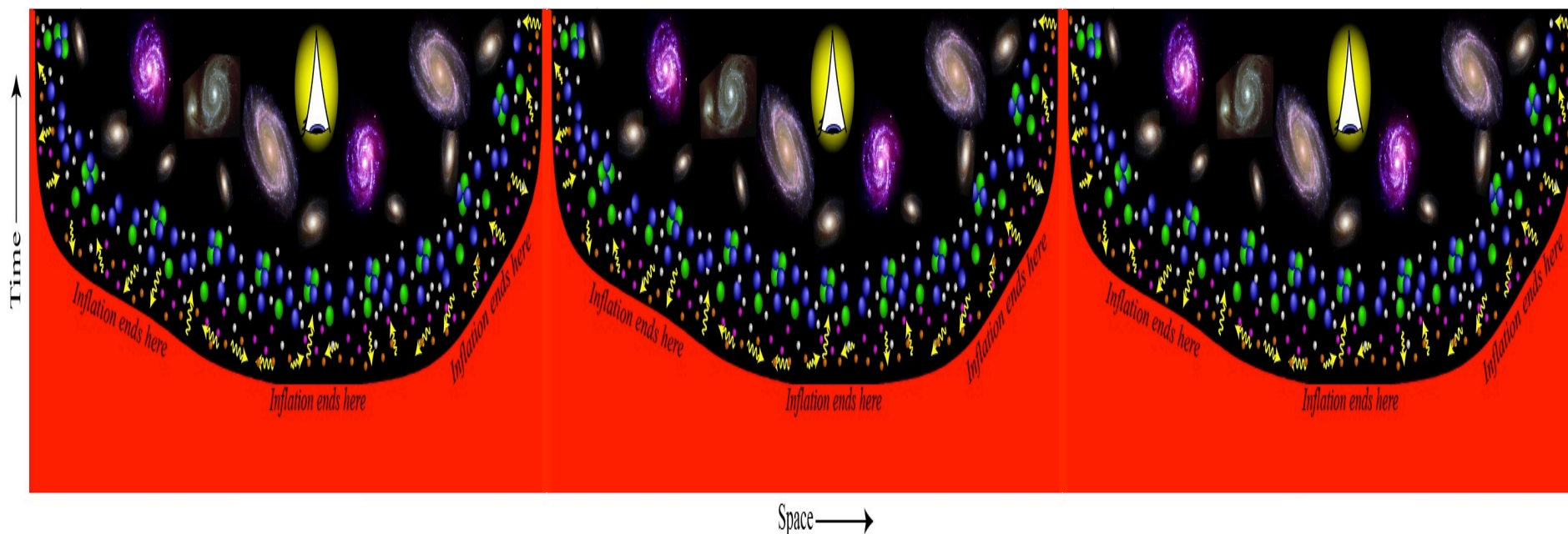


Summary of
Lecture 2

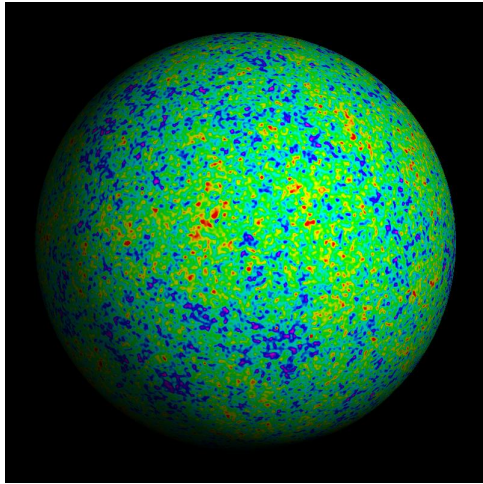


Summary of Lecture 2

Inflation can start with a finite region of 3D space and make many separate infinite universes within it:



Multiverse Level I: Regions beyond our cosmic horizon



Giordano Bruno (executed 1600)
Ellis & Brundrit 1979, Q.J.R. Astr. Soc. 20, 37
Garriga & Vilenkin 2001, Phys.Rev. D64, 043511

- Size of our universe $\sim 10^{26}$ m
- Closest identical universe $\sim 10^{10^{118}}$ m

Features:

- Same (effective) laws of physics
- Different initial conditions

Assumptions:

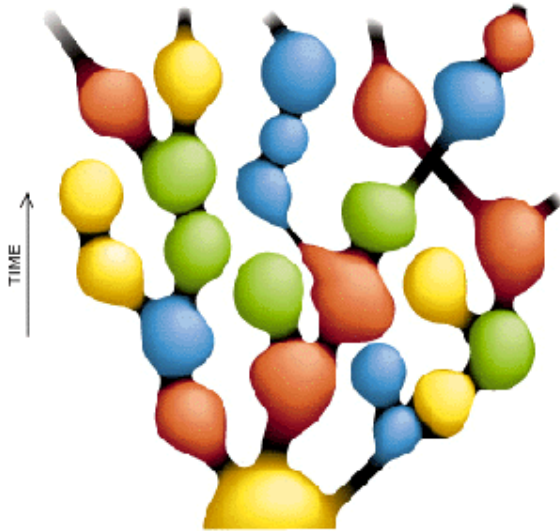
- Infinite space
- Ergodic matter distribution

Evidence:

- Microwave background measurements point to flat, infinite space, large-scale smoothness
- Simplest model
- Data supports inflation, which supports both assumptions

Summary of
Lecture 2

Multiverse Level II: Other post-inflation bubbles (other “big bangs” in same 3D space)



Giordano Bruno (executed 1600)
Ellis & Brundrit 1979, Q.J.R. Astr. Soc. 20, 37
Garriga & Vilenkin 2001, Phys.Rev. D64, 043511

Features:

- Size of our universe $\sim 10^{26}$ m
- Closest identical universe $\sim 10^{10^{118}}$ m
- Same fundamental equations of physics, but maybe different constants, particles and dimensionality

Assumptions:

- Inflation happened
- Inflation is eternal (it generically is)

Evidence:

- Inflation explains flat space, near scale-invariant fluctuations, the “bang” in our Big Bang and solves horizon and monopole problems.
- Explains fine-tuned parameters like dark energy density

Summary of
Lecture 2

PHYSICS OR PHILOSOPHY?

Q: Are theories which predict the existence of unobservable parallel universes untestable?



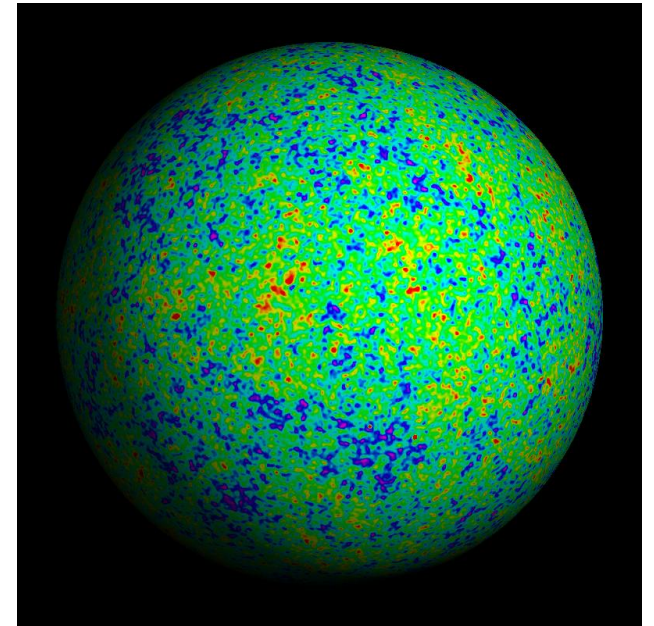
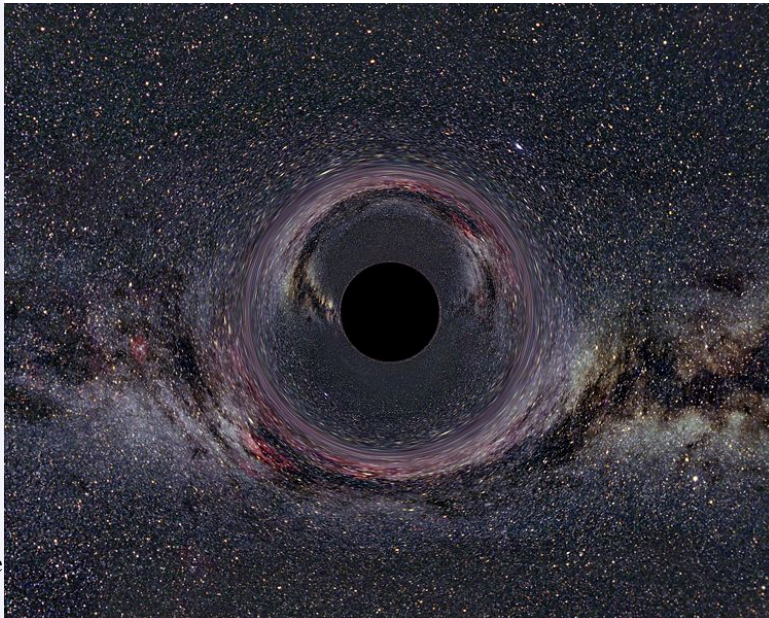
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May 13, 2011

PHYSICS OR PHILOSOPHY?

Q: Are theories which predict the existence of unobservable parallel universes untestable?

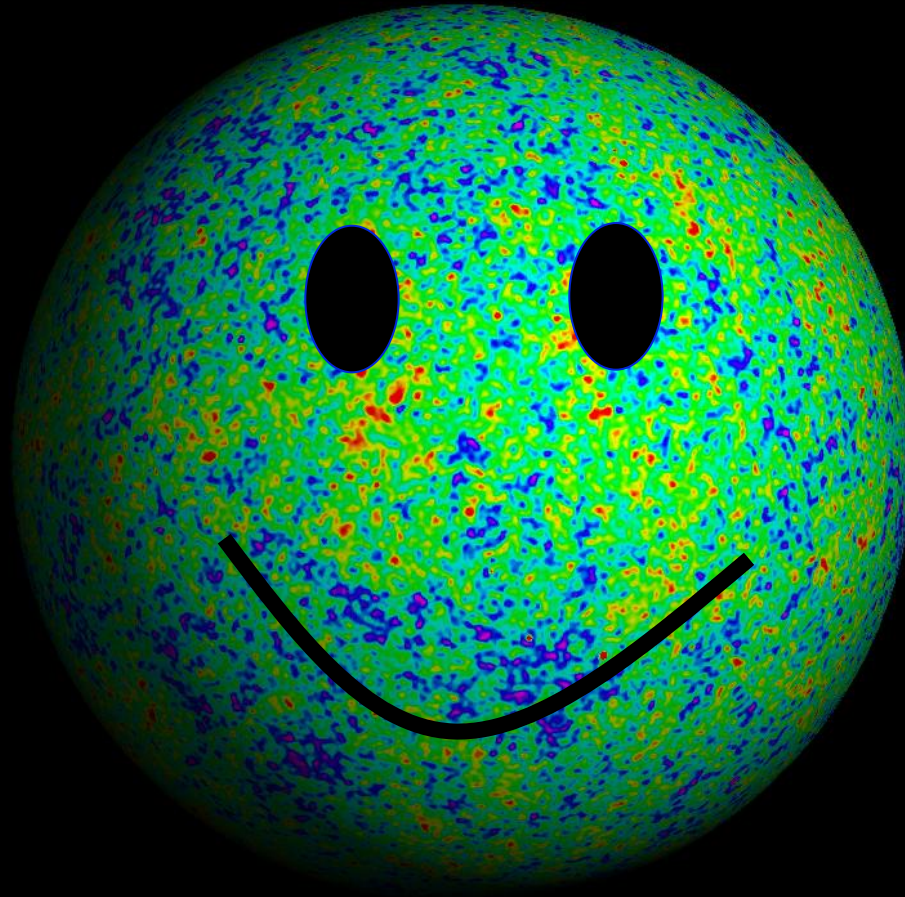
A: No, as long as they also make predictions for things we *can* observe.

Parallel universes are not a theory, but the prediction of certain theories.



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**But can we understand all this in terms of a fundamental
“theory of everything”?**



gr-qc/9704009
arXiv:0704.06462
arXiv: 0709.4024

Can we describe reality purely mathematically?

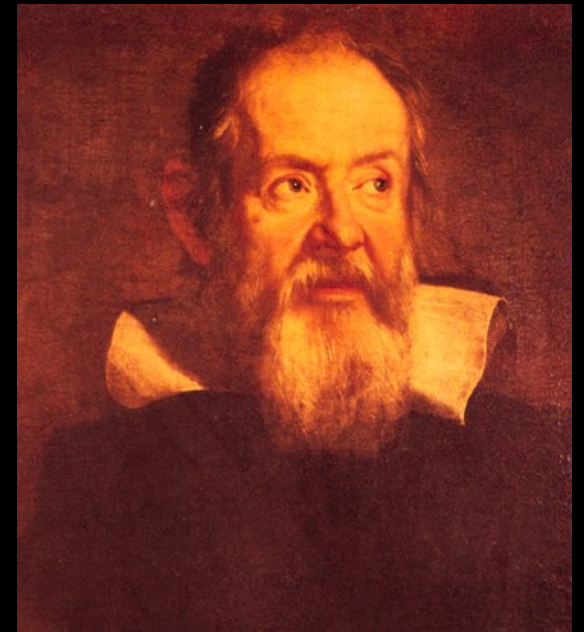
Outline:

- Nature seems unreasonably well described by math
 - What do we mean by this?
 - What do I think it means?
- The Mathematical Universe Hypothesis
 - What it means
 - What it implies
 - The illusion of change, randomness, etc.
 - The Level IV multiverse
- Evidence? Arguments pro and con?



Math, math everywhere!

Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the characters in which it is written. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these one is wandering in a dark labyrinth.



Galileo Galilei 1623

But where's the
math?

Shapes

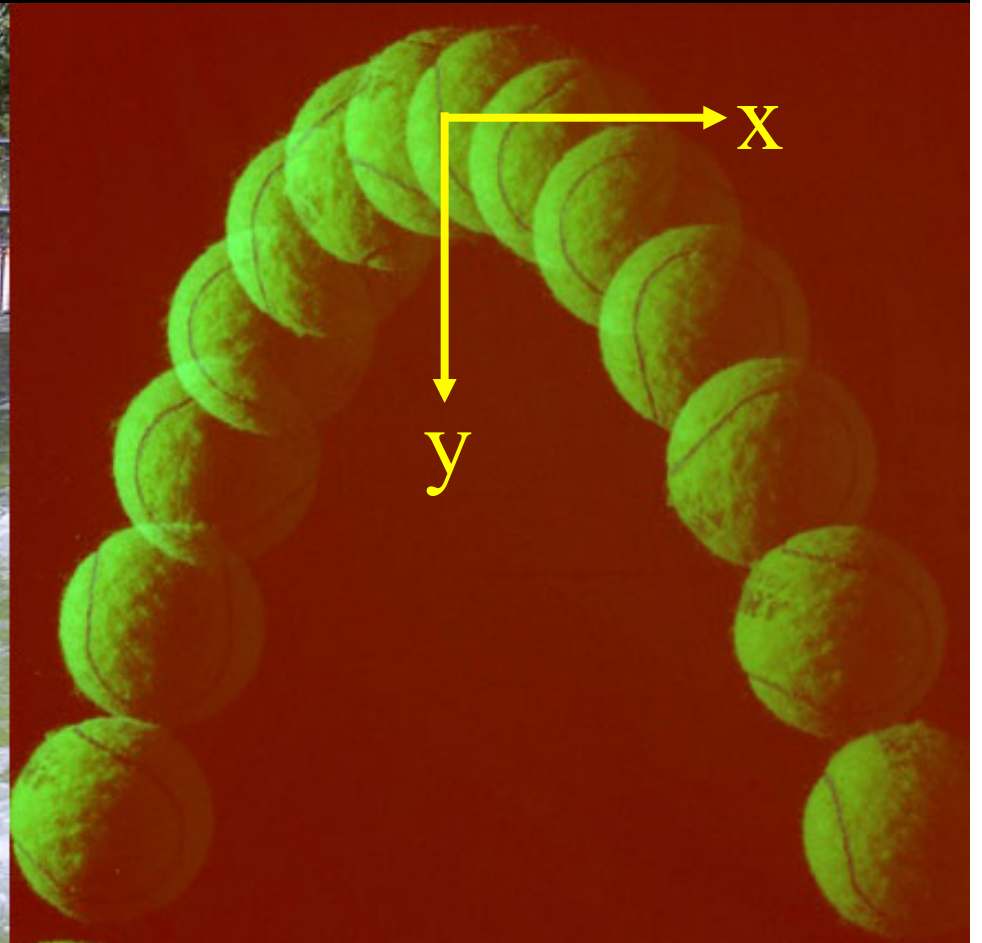
&

patterns

Nature is full of recurrent patterns and shapes!

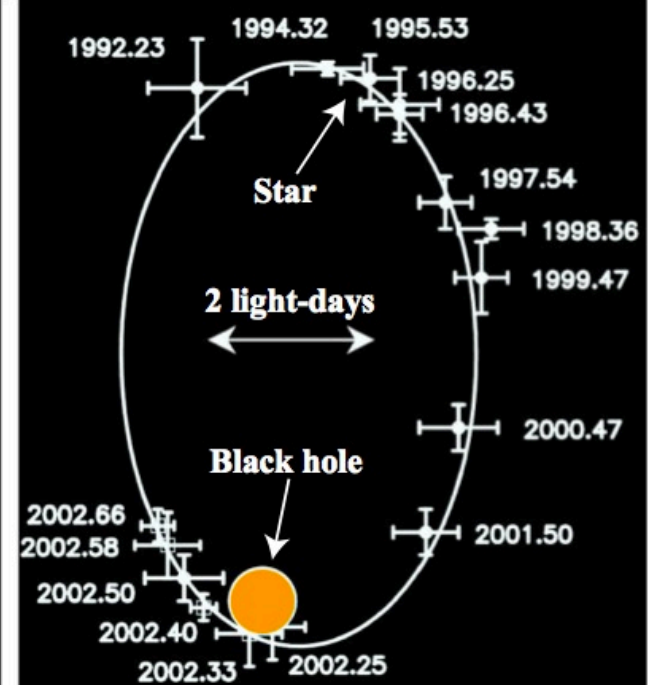
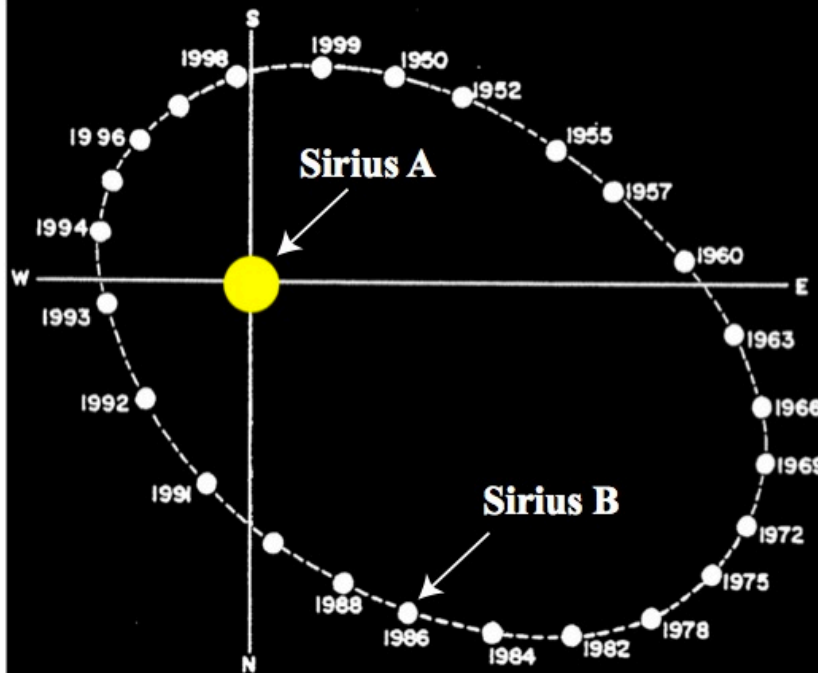
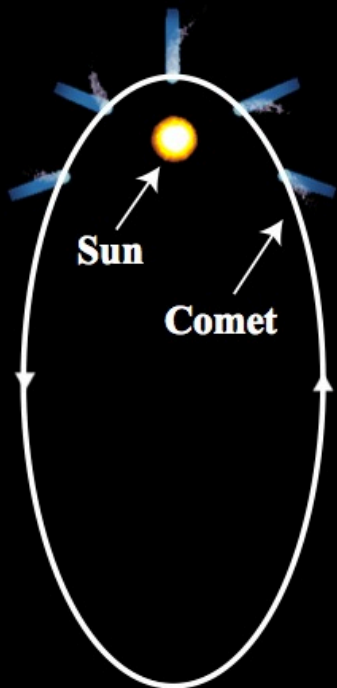


Nature is full of recurrent patterns and shapes!

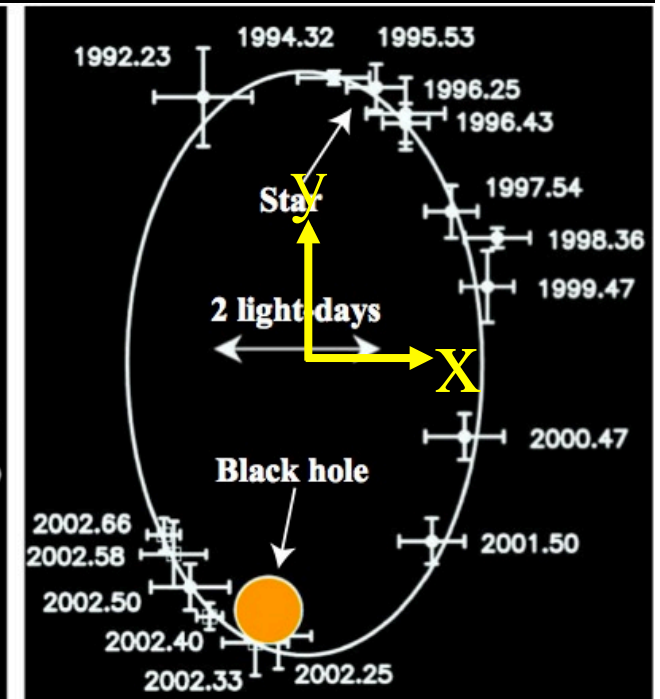
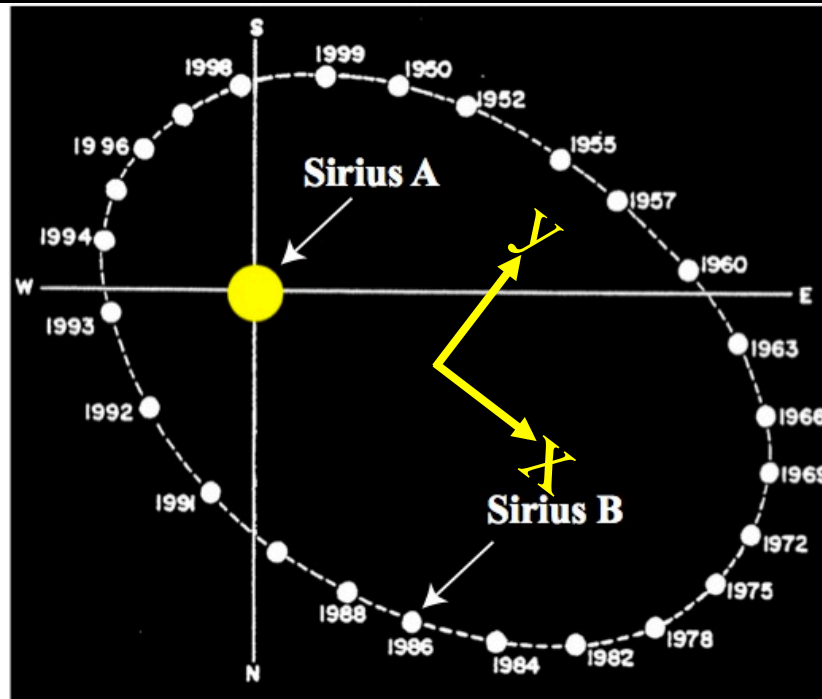
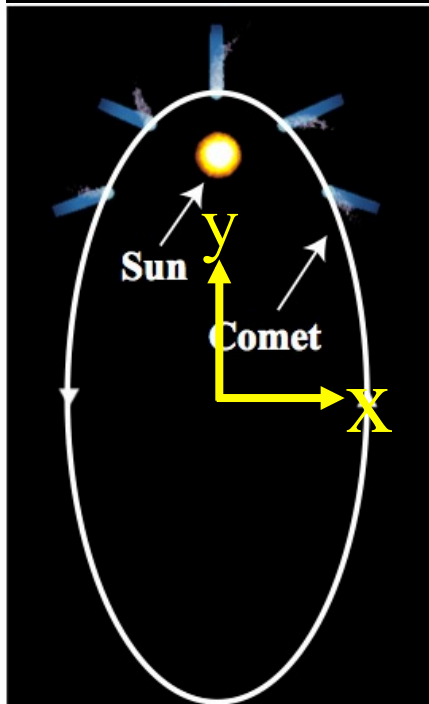


Parabola: $y = x^2$

Nature is full of recurrent patterns and shapes!



Nature is full of recurrent patterns and shapes!



Ellipse:
$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\dot{\mathbf{B}} \\ \nabla \times \mathbf{B} &= \dot{\mathbf{E}} + \mathbf{J}\end{aligned}$$

Maxwell 1862

$$\begin{pmatrix} ct \\ x \end{pmatrix} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \begin{pmatrix} 1 & \frac{v}{c} \\ \frac{v}{c} & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \end{pmatrix}$$

$$E = mc^2$$

Einstein 1905

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

$$\ddot{x}^\lambda = \Gamma_{\mu\nu}^\lambda \dot{x}^\mu \dot{x}^\nu$$

Einstein 1915

$$i\hbar \frac{d}{dt} |\psi\rangle = H |\psi\rangle$$

Schrödinger 1926

$$d\tau^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}$$

Friedmann 1922

?

You 2017?

$$E = mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$E = \gamma mc^2$$

$$p = \gamma mv$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$$

$$\dot{x}^2 = \int_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

$$\exp\left[\frac{i}{\hbar} \int \sqrt{-g} [R + L_{\text{matter}}] d^4x\right]$$

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = H |\psi\rangle$$

$$\nabla \cdot \vec{E} = \rho$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\dot{\vec{B}}$$

$$\nabla \times \vec{B} = \vec{J} + \dot{\vec{E}}$$

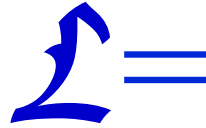


Numbers

So what *does* go
on the T-shirt?

$$\begin{aligned}
& -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e + \\
& \frac{1}{2}ig_s^2 (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a + \bar{G}^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- - \\
& M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H - \\
& \frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+ \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w} M \phi^0 \phi^0 - \beta_h \left[\frac{2M^2}{g^2} + \right. \\
& \left. \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right] + \frac{2M^4}{g^2} \alpha_h - igc_w [\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\mu^0 (W_\nu^+ \partial_\nu W_\mu^- - \\
& W_\nu^- \partial_\nu W_\mu^+)] - ig s_w [\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - \\
& W_\mu^- \partial_\nu W_\mu^+) + A_\mu (W_\nu^+ \partial_\nu W_\mu^- - W_\nu^- \partial_\nu W_\mu^+)] - \frac{1}{2}g^2 W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- + \\
& \frac{1}{2}g^2 W_\mu^+ W_\nu^- W_\mu^+ W_\nu^- + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - Z_\mu^0 Z_\nu^0 W_\mu^+ W_\nu^-) + \\
& g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\mu^+ W_\nu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
& W_\nu^+ W_\mu^-) - 2A_\mu Z_\mu^0 W_\nu^+ W_\nu^-] - g\alpha [H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-] - \\
& \frac{1}{8}g^2 \alpha_h [H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2] - \\
& gM W_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w} Z_\mu^0 Z_\mu^0 H - \frac{1}{2}ig [W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - \\
& W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)] + \frac{1}{2}g [W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) - W_\mu^- (H \partial_\mu \phi^+ - \\
& \phi^+ \partial_\mu H)] + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \\
& ig s_w M A_\mu (W_\mu^+ \phi^- - W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + \\
& ig s_w A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \frac{1}{4}g^2 W_\mu^+ W_\mu^- [H^2 + (\phi^0)^2 + 2\phi^+ \phi^-] - \\
& \frac{1}{4}g^2 \frac{1}{c_w} Z_\mu^0 Z_\mu^0 [H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-] - \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
& W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda \gamma \partial \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma \partial + \\
& m_d^\lambda) d_j^\lambda + ig s_w A_\mu [-(\bar{e}^\lambda \gamma e^\lambda) + \frac{2}{3}(\bar{u}_j^\lambda \gamma u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma d_j^\lambda)] + \frac{ig}{4c_w} Z_\mu^0 [(\bar{\nu}^\lambda \gamma^\mu (1 + \\
& \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (\frac{4}{3}s_w^2 - 1 - \gamma^5) u_j^\lambda) + \\
& (\bar{d}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 - \gamma^5) d_j^\lambda)] + \frac{ig}{2\sqrt{2}} W_\mu^+ [(\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) e^\lambda) + (\bar{u}_j^\lambda \gamma^\mu (1 + \\
& \gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1 + \gamma^5) u_j^\lambda)] + \\
& \frac{ig}{2\sqrt{2}} \frac{m_\lambda^1}{M} [-\phi^+ (\bar{\nu}^\lambda (1 - \gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1 + \gamma^5) \nu^\lambda)] - \frac{g}{2} \frac{m_\lambda^1}{M} [H (\bar{e}^\lambda e^\lambda) + \\
& i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \\
& \gamma^5) d_j^\kappa)] + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\lambda) - m_u^\lambda (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\lambda) - \\
& \frac{g}{2} \frac{m_\lambda^1}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_\lambda^1}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda^1}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda^1}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \\
& \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + \\
& igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + ig s_w W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + \\
& igc_w W_\mu^- (\partial_\mu \bar{X}^- X^0 - \partial_\mu \bar{X}^0 X^+) + ig s_w W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + \\
& igc_w Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) + ig s_w A_\mu (\partial_\mu \bar{X}^+ X^+ - \partial_\mu \bar{X}^- X^-) - \\
& \frac{1}{2}gM [\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w} \bar{X}^0 X^0 H] + \frac{1-2c_w^2}{2c_w} igM [\bar{X}^+ X^0 \phi^+ - \\
& \bar{X}^- X^0 \phi^-] + \frac{1}{2c_w} igM [\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-] + igM s_w [\bar{X}^0 X^- \phi^+ - \\
& \bar{X}^0 X^+ \phi^-] + \frac{1}{2}igM [\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0]
\end{aligned}$$

The Standard Model Lagrangian



(From T.D. Gutierrez)

Why these values?

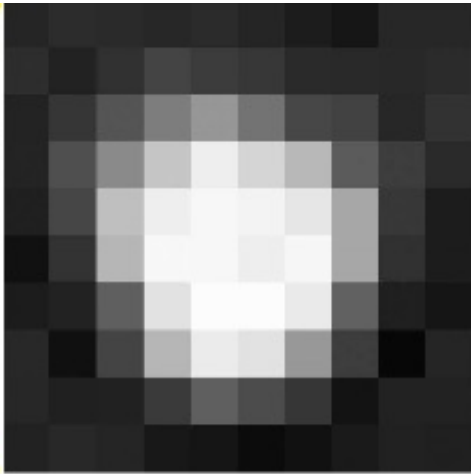
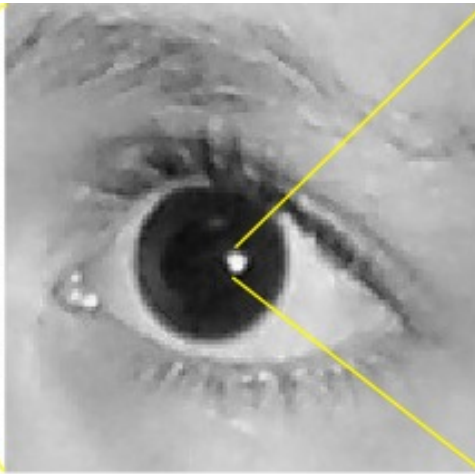
Standard model parameters:

Particle physics

Cosmology

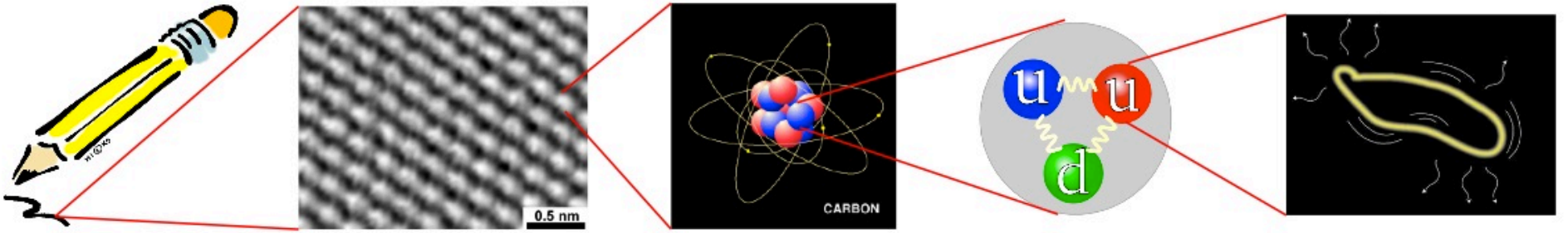
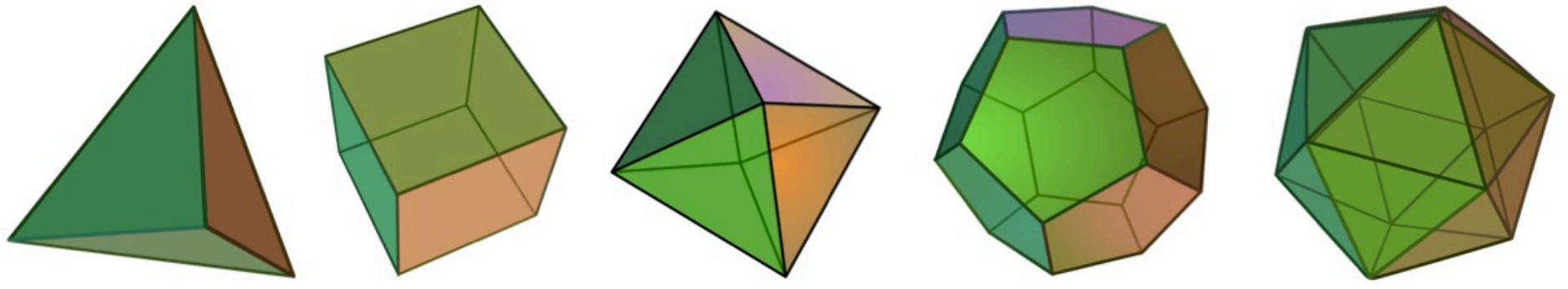
Parameter	Meaning	Measured value
g	Weak coupling constant at m_Z	0.6520 ± 0.0001
θ_W	Weinberg angle	0.48290 ± 0.00005
g_s	Strong coupling constant at m_Z	1.221 ± 0.022
μ^2	Quadratic Higgs coefficient	$\sim -10^{-33}$
λ	Quartic Higgs coefficient	$\sim 1?$
G_e	Electron Yukawa coupling	2.94×10^{-6}
G_μ	Muon Yukawa coupling	0.000607
G_τ	Tauon Yukawa coupling	0.0102156233
G_u	Up quark Yukawa coupling	0.000016 ± 0.000007
G_d	Down quark Yukawa coupling	0.00003 ± 0.00002
G_c	Charm quark Yukawa coupling	0.0072 ± 0.0006
G_s	Strange quark Yukawa coupling	0.0006 ± 0.0002
G_t	Top quark Yukawa coupling	1.002 ± 0.029
G_b	Bottom quark Yukawa coupling	0.026 ± 0.003
$\sin \theta_{12}$	Quark CKM matrix angle	0.2243 ± 0.0016
$\sin \theta_{23}$	Quark CKM matrix angle	0.0413 ± 0.0015
$\sin \theta_{13}$	Quark CKM matrix angle	0.0037 ± 0.0005
δ_{13}	Quark CKM matrix phase	1.05 ± 0.24
θ_{qcd}	CP-violating QCD vacuum phase	$< 10^{-9}$
G_{ν_e}	Electron neutrino Yukawa coupling	$< 1.7 \times 10^{-11}$
G_{ν_μ}	Muon neutrino Yukawa coupling	$< 1.1 \times 10^{-6}$
G_{ν_τ}	Tau neutrino Yukawa coupling	< 0.10
$\sin \theta'_{12}$	Neutrino MNS matrix angle	0.55 ± 0.06
$\sin 2\theta'_{23}$	Neutrino MNS matrix angle	≥ 0.94
$\sin \theta'_{13}$	Neutrino MNS matrix angle	≤ 0.22
δ'_{13}	Neutrino MNS matrix phase	?
ρ_Λ	Dark energy density	$(1.25 \pm 0.25) \times 10^{-123}$
ξ_b	Baryon mass per photon ρ_b/n_γ	$(0.50 \pm 0.03) \times 10^{-28}$
ξ_c	Cold dark matter mass per photon ρ_c/n_γ	$(2.5 \pm 0.2) \times 10^{-28}$
ξ_ν	Neutrino mass per photon $\rho_\nu/n_\gamma = \frac{3}{11} \sum m_{\nu_i}$	$< 0.9 \times 10^{-28}$
Q	Scalar fluctuation amplitude δ_H on horizon	$(2.0 \pm 0.2) \times 10^{-5}$
n_s	Scalar spectral index	0.98 ± 0.02

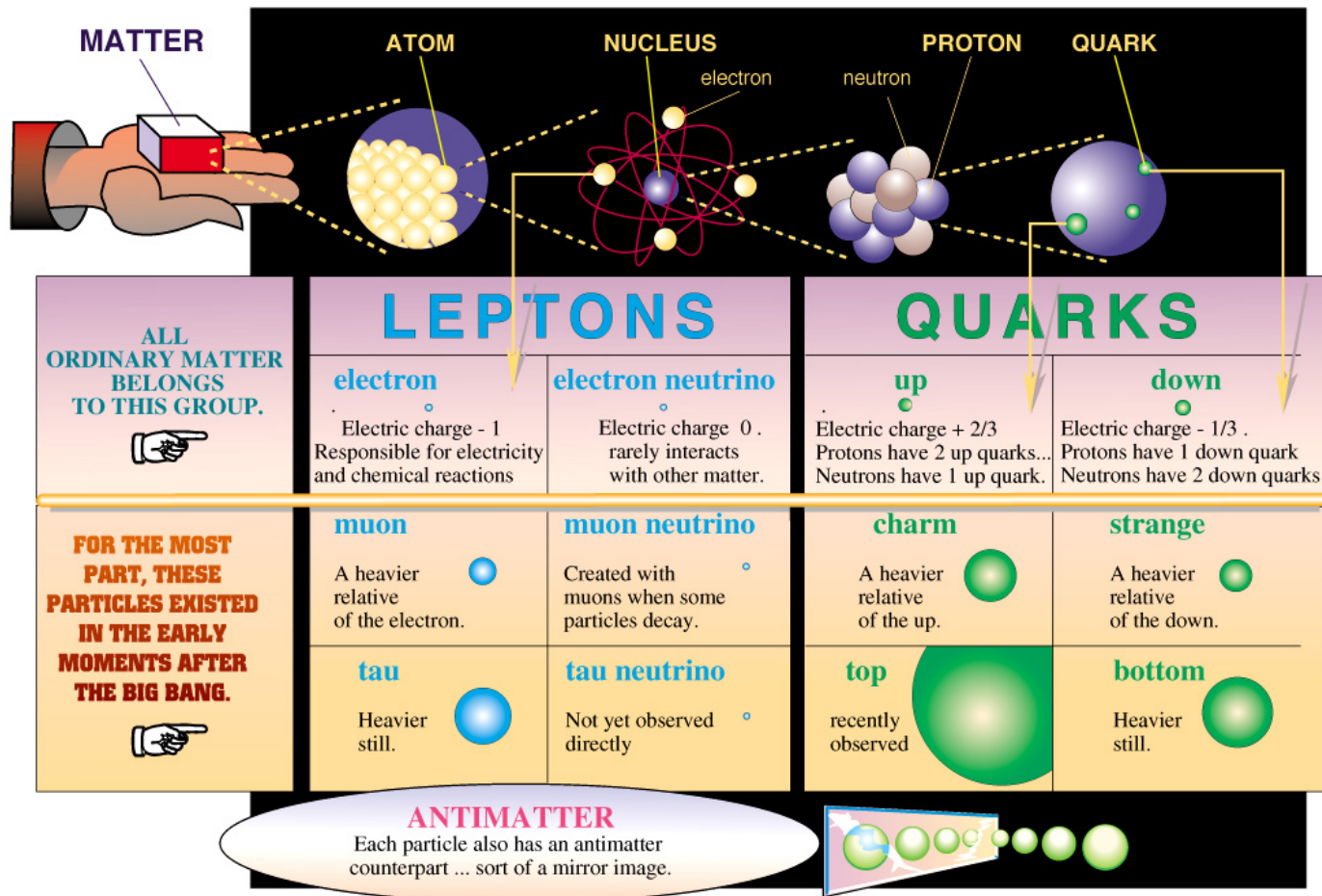
$$C = h = G = k_b = q_e = 1$$



25	30	29	27	30	25	20	16	27	27
31	24	33	50	42	39	29	28	28	30
24	38	64	103	129	94	51	48	27	35
26	59	115	183	232	202	166	69	43	29
22	50	176	235	243	238	222	148	40	19
12	36	166	244	245	233	242	148	33	20
19	23	73	218	251	251	228	75	21	16
28	12	50	164	230	217	132	43		25
27	22	23	41	73	56	38	14	23	23
24	27	26	17	15	10	13	19	23	22

“Stuff” seems to be
made of
mathematical objects





CERN AC _ E11-7

Particle name	Mass in MeV	Charge	Spin	Isospin	Baryon number	Lepton number
Proton	938.3	1	1/2	1/2	1	0
Neutron	939.6	0	1/2	1/2	1	0
Electron	0.511	-1	1/2	-1/2	0	1
Up quark	1.5-4	2/3	1/2	1/2	1/3	0
Down quark	4-8	-1/3	1/2	-1/3	1/3	0
Electron neutrino	$< 10^{-6}$	0	1/2	1/2	0	1
Photon	0	0	1	0	0	0

Space also seems to
have only
mathematical
properties

External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.



Mathematical Universe Hypothesis (MUH):

Or external physical reality is a mathematical structure.



External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.



External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.



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May 13, 2011

External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.



TOE



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External Reality Hypothesis (ERH):

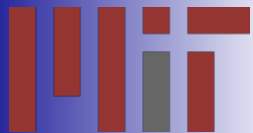
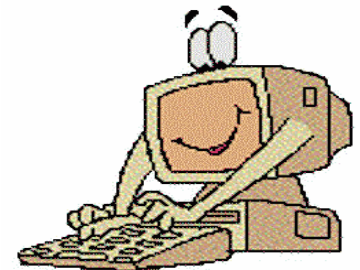
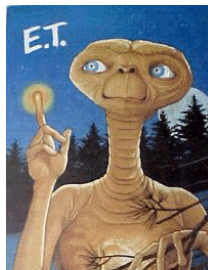
There exists an external physical reality completely independent of us humans.



TOE



For this description of the external physical reality to be *complete*, it must be devoid of human “baggage”.



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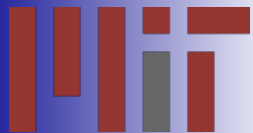
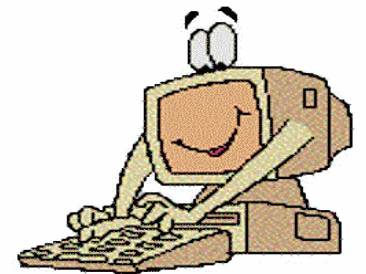
External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.

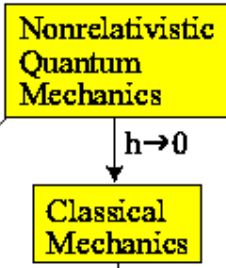


TOE

We humans have a common understanding of words like “object”, “experiment”, “observation”, “cause”, “particle”, “string”, but computers don't!



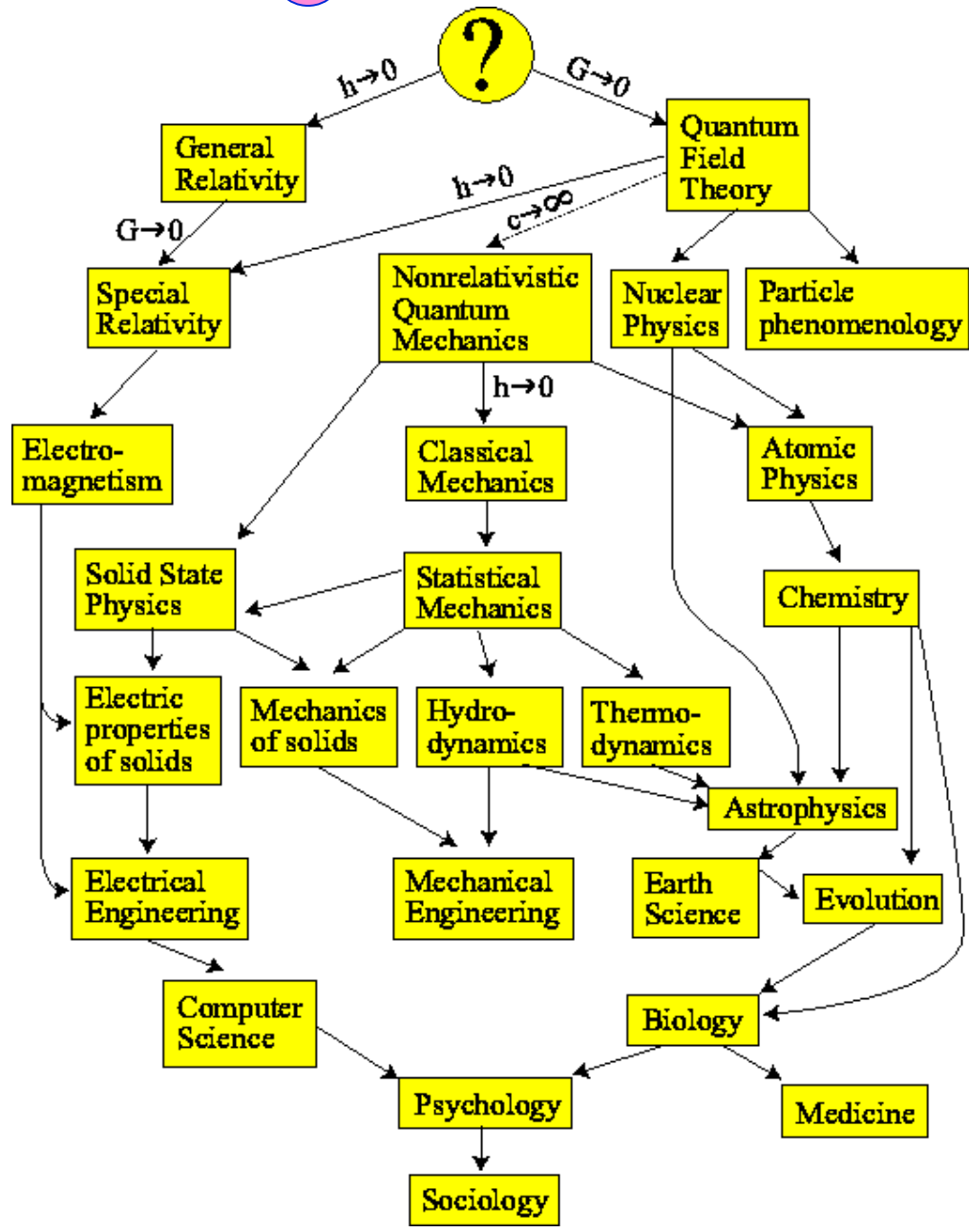
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Tegmark & Wheeler 2001, quant-ph/0101077



Less baggage

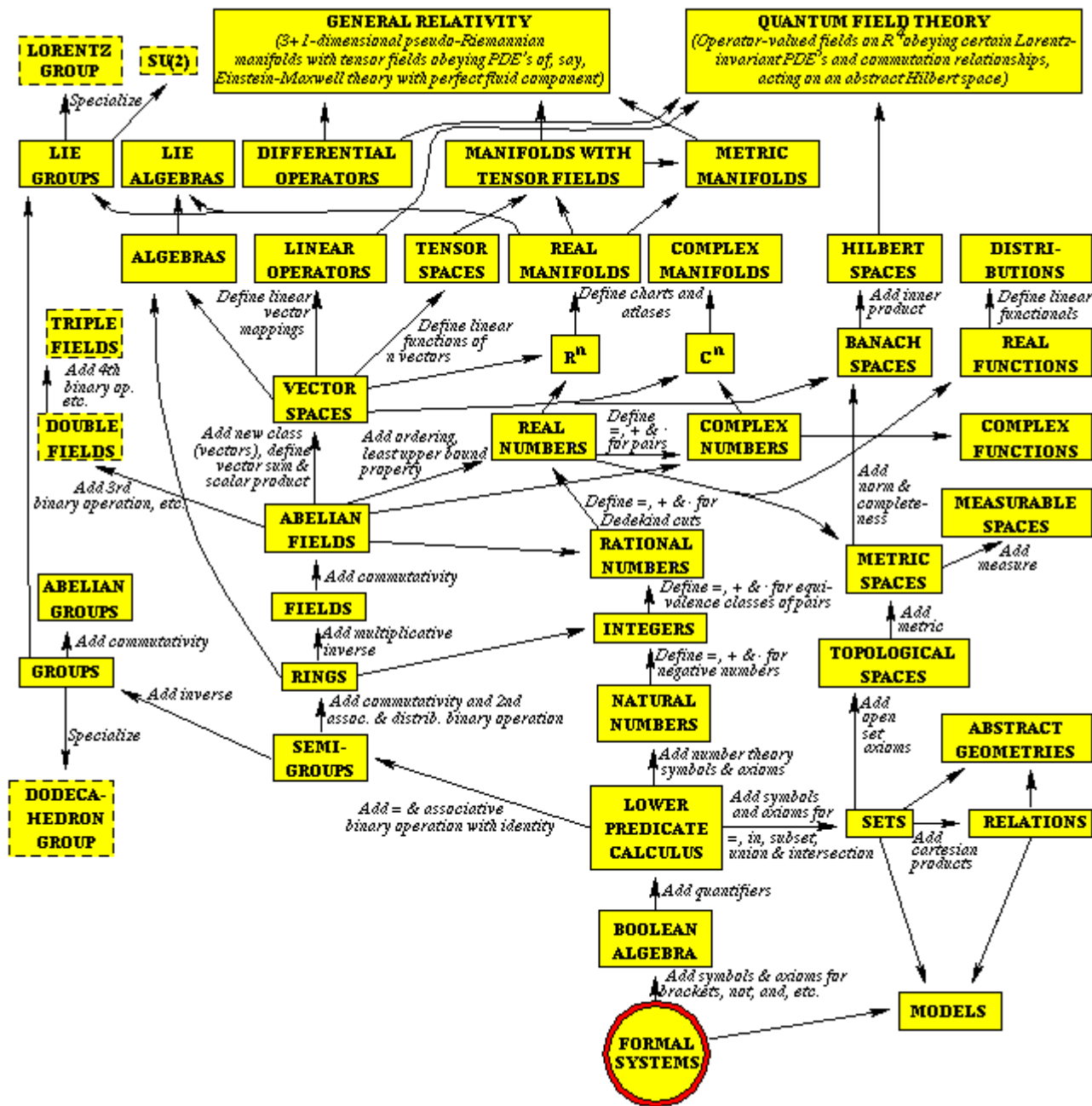


More baggage



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A mathematical structure: abstract entities with relations between them



External Reality Hypothesis (ERH):

There exists an external physical reality completely independent of us humans.



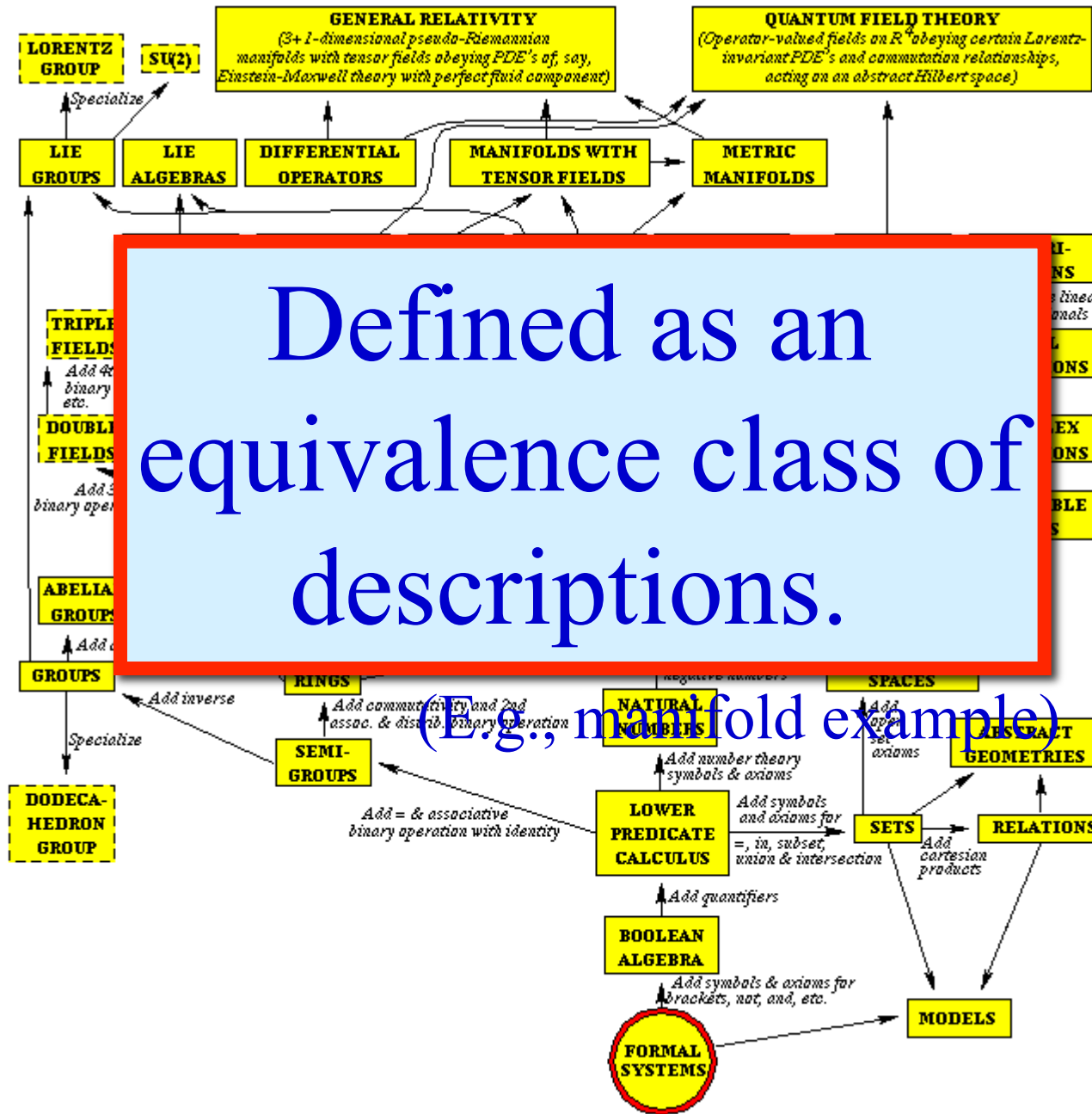
Mathematical Universe Hypothesis (MUH):

Or external physical reality is a mathematical structure.



What's a
mathematical
structure?

A mathematical structure: abstract entities with relations between them



Tegmark 1997, gr-qc/9704009, Ann. Phys. 270, 151



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Less abstract
More baggage

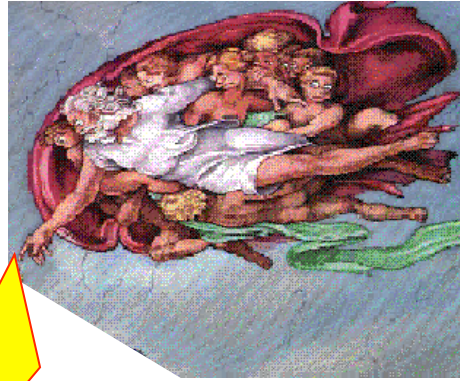


More abstract
Less baggage



1.e4 e5 2.f4 exf4 3.Bc4 Qh4+
4.Kf1 b5 5.Bxb5 Nf6 6.Nf3 Qh6
7.d3 Nh5 8.Nh4 Qg5 9.Nf5 c6
10.g4 Nf6 11.Rg1 cxb5 12.h4 Qg6
13.h5 Qg5 14.Qf3 Ng8 15.Bxf4 Qf6
16.Nc3 Bc5 17.Nd5 Qxb2
18.Bd6 Bxg1 19. e5 Qxa1+
20. Ke2 Na6 21.Nxg7+ Kd8
22.Qf6+ Nxf6 23.Be7

So is “*God created Heaven and Earth*” a mathematical structure?



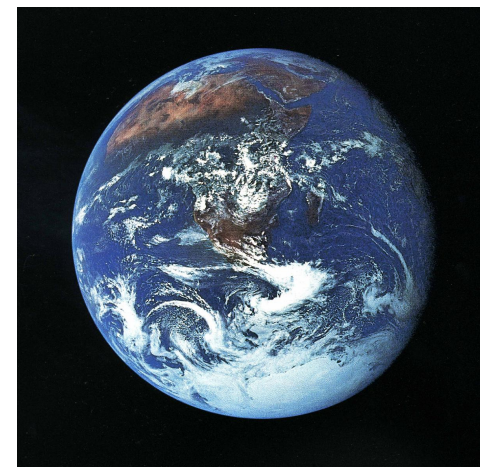
God

created

created



Heaven

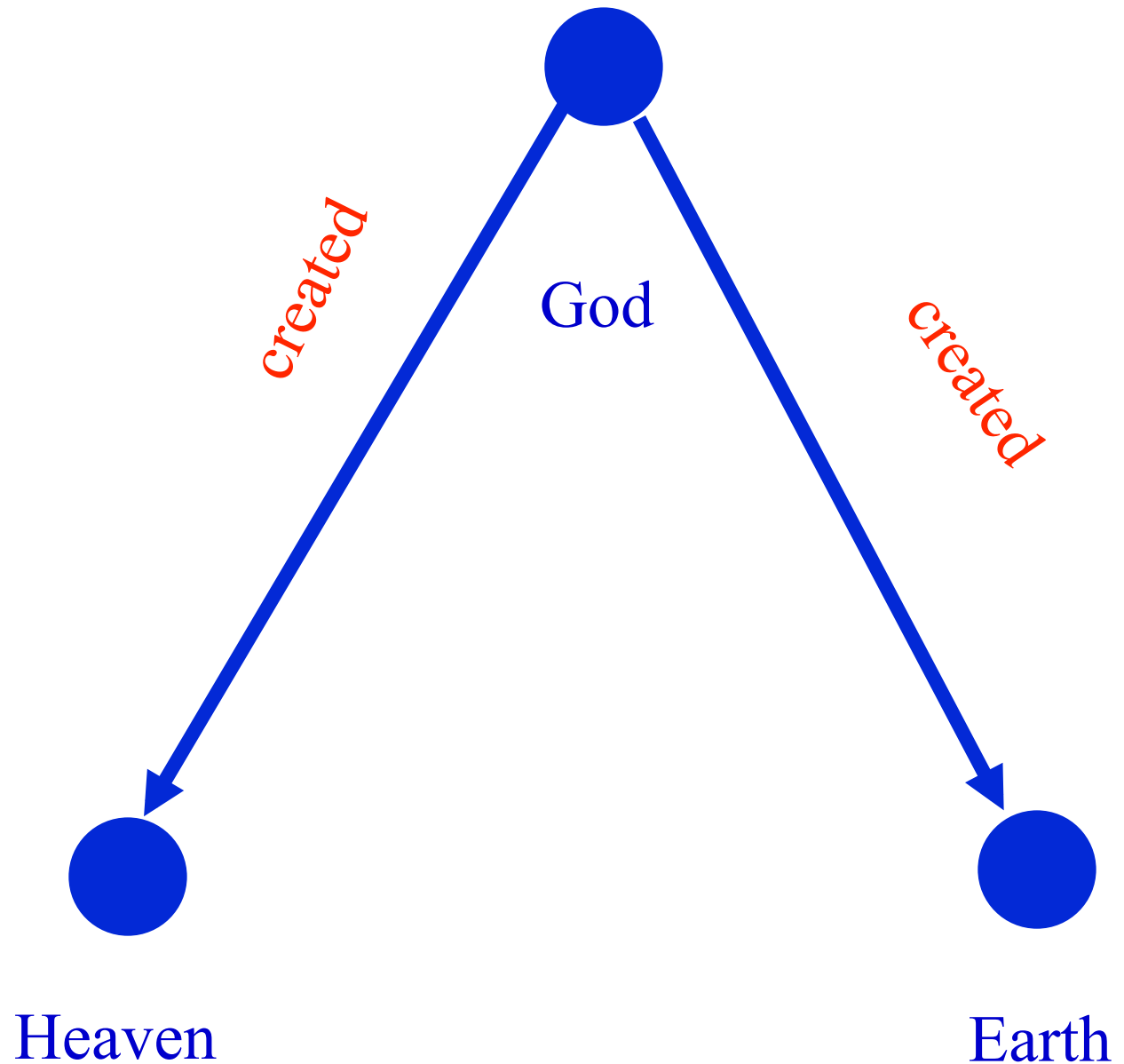


Earth



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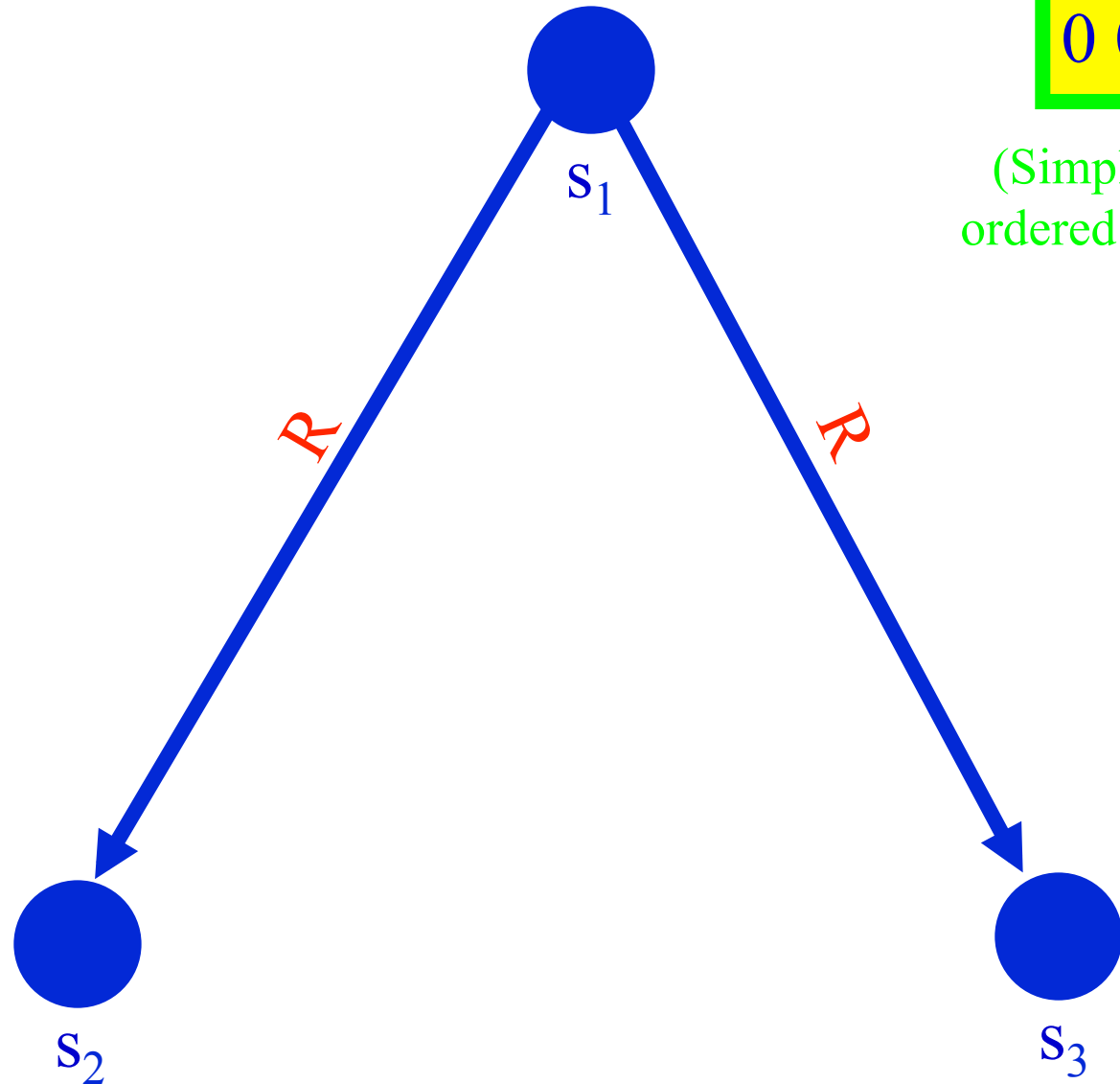
“God created Heaven and Earth”:



“ $s_1 R s_2$ & $s_1 R s_3$ ”:

0	1	1
0	0	0
0	0	0

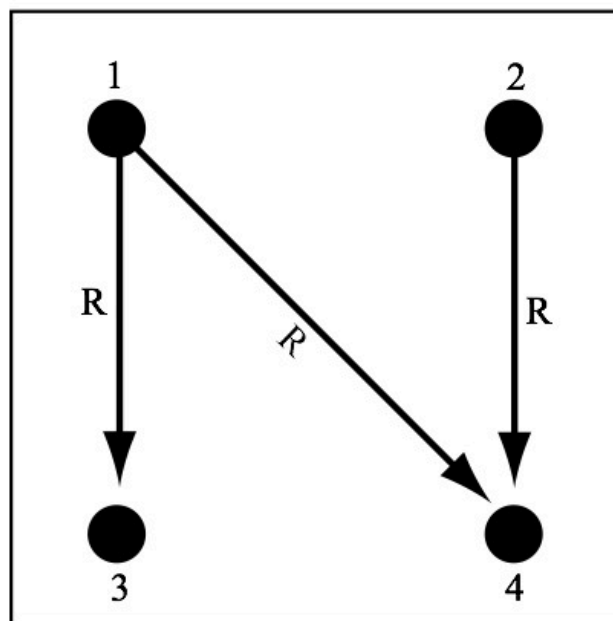
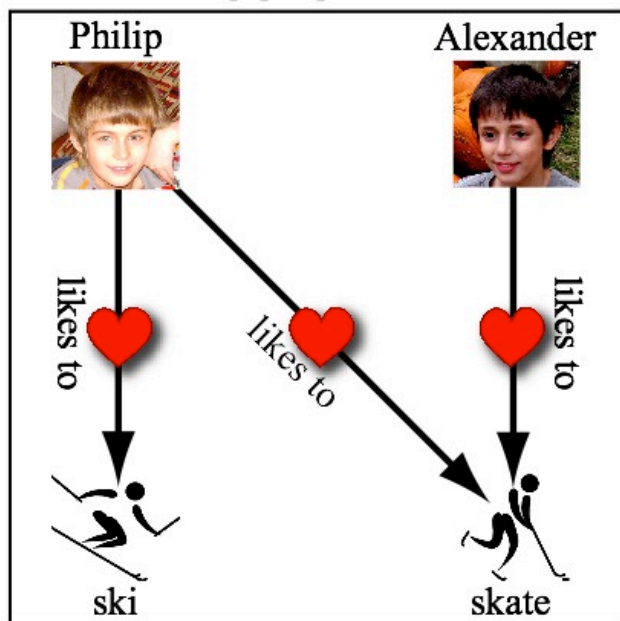
(Simply an ordered graph)



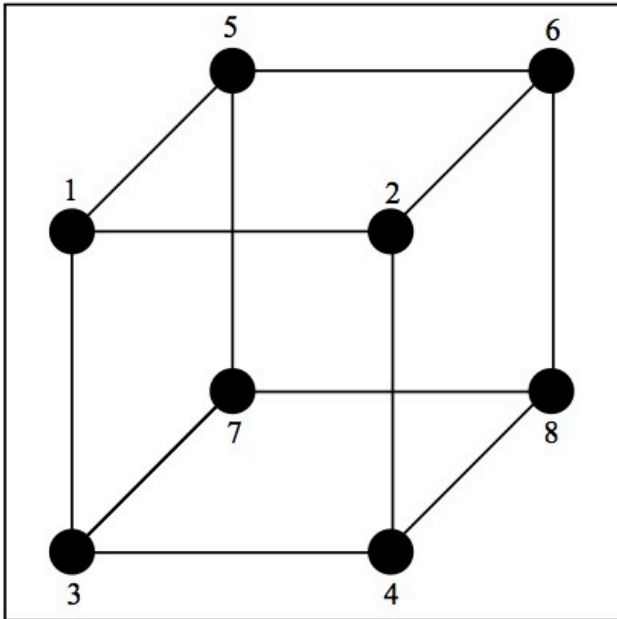
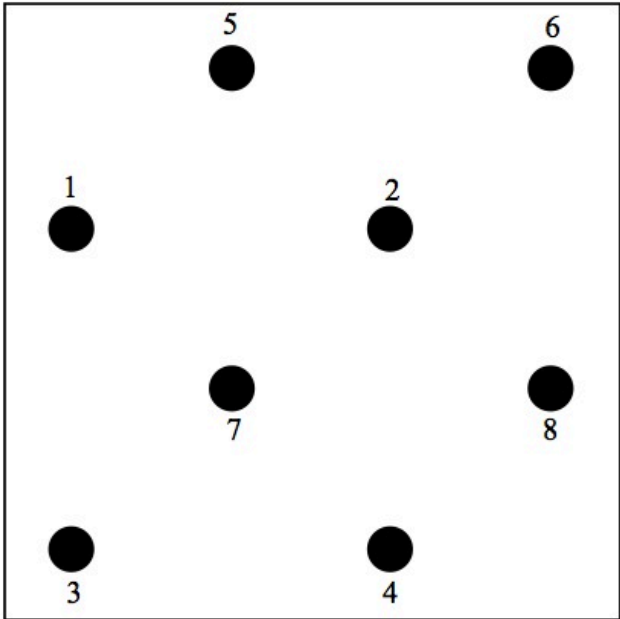
Less abstract
More baggage



More abstract
Less baggage



	1	2	3	4
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0
4	0	0	0	0



	1	2	3	4	5	6	7	8
1	0	1	1	0	1	0	0	0
2	1	0	0	1	0	1	0	0
3	1	0	0	1	0	0	1	0
4	0	1	1	0	0	0	0	1
5	1	0	0	0	0	1	1	0
6	0	1	0	0	1	0	0	1
7	0	0	1	0	1	0	0	1
8	0	0	0	1	0	1	1	0

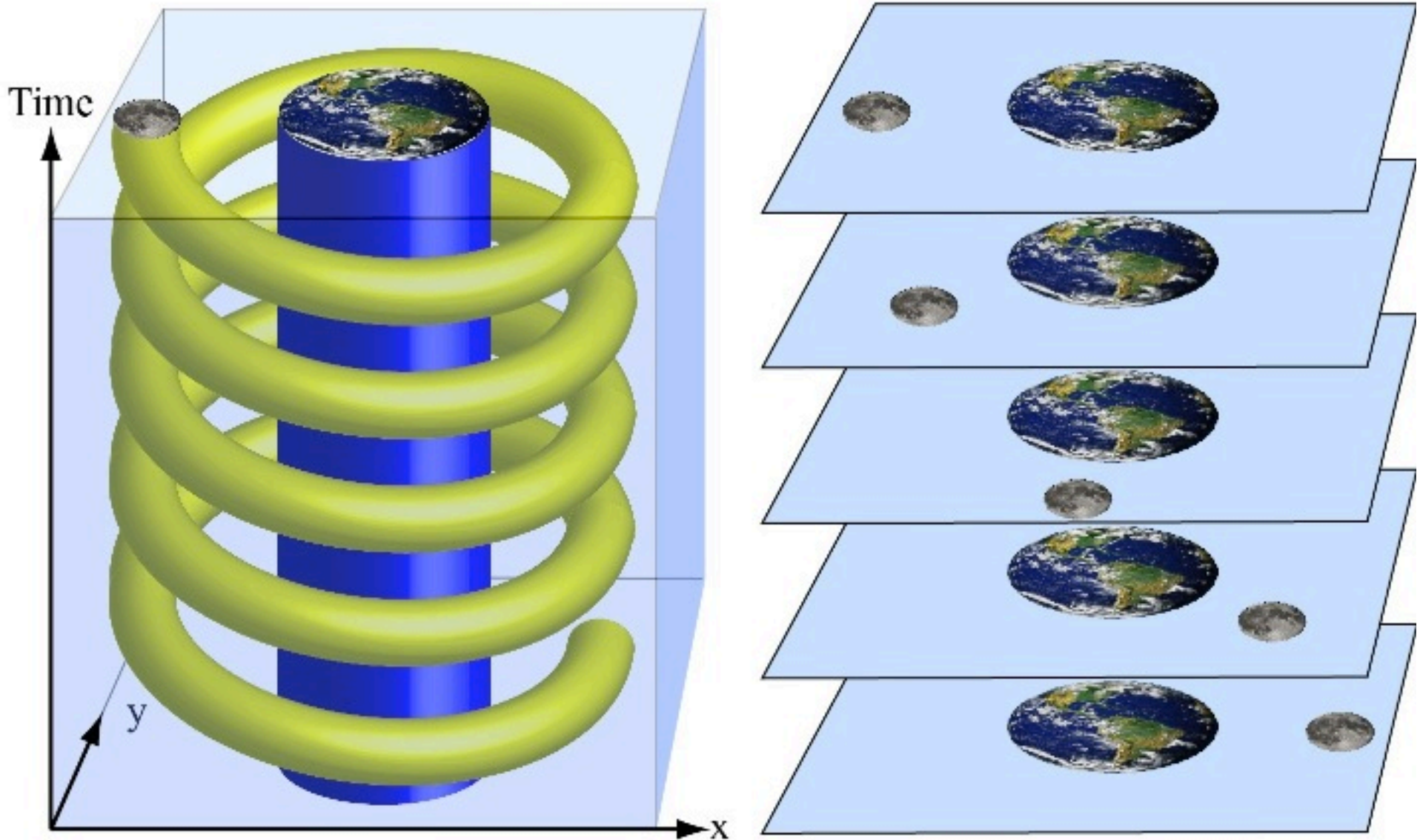
The bottom line:

- Since antiquity, people have puzzled over why our physical world can be so accurately described by mathematics.
- Ever since, physicists have kept discovering more shapes, patterns and regularities in nature that are describable by mathematical equations.
- The fabric of our physical reality contains dozens of pure numbers, from which all measured constants can in principle be calculated.
- Some key physical entities like empty space, elementary particles, and the wavefunction, appear to be purely mathematical in the sense that their only intrinsic properties are mathematical properties.
- The *External Reality Hypothesis* (ERH) — that there exists an external physical reality completely independent of us humans — is accepted by most but not all physicists.
- With a sufficiently broad definition of mathematics, this implies the *Mathematical Universe Hypothesis* (MUH) that our physical world is a mathematical structure.
- This means that our physical world not only is *described* by mathematics, but that it *is* mathematical (a mathematical structure), making us self-aware parts of a giant mathematical object.
- A mathematical structure is an abstract set of entities with relations between them. The entities have no “baggage”: they have no properties whatsoever except these relations.

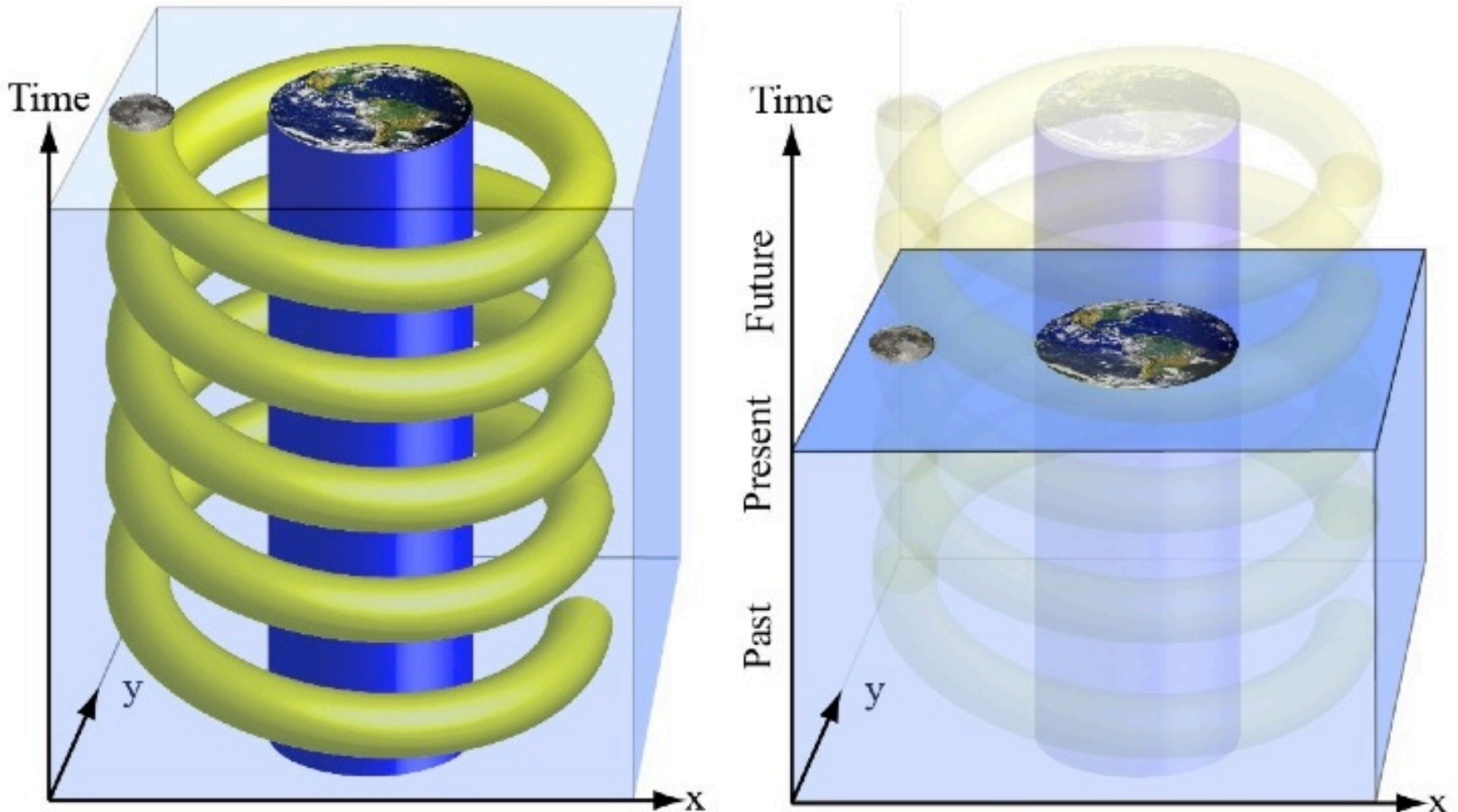


Implications

The flow of time is an illusion:

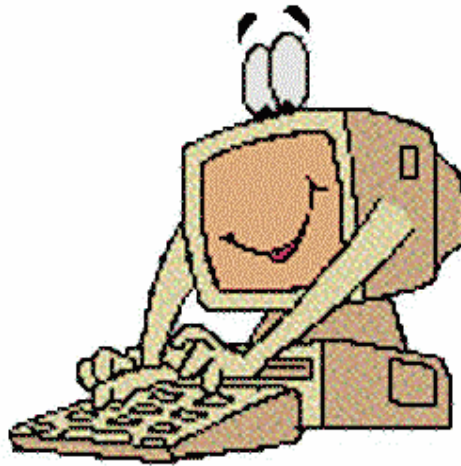


Change is an illusion,
the past never disappears:



Simulation
hypothesis
implications

Do we live in a simulation? (The Matrix, Tipler, Bostrom, Schmidhuber, Lloyd...)



- McCabe, Penrose: no, at least not a digital one
- Involves measure problem



```

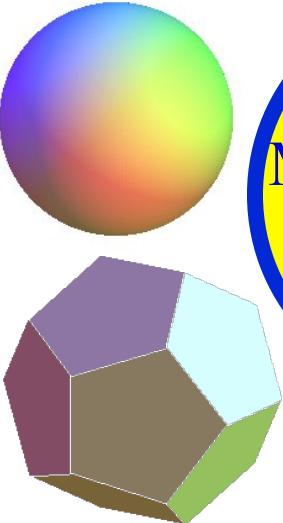
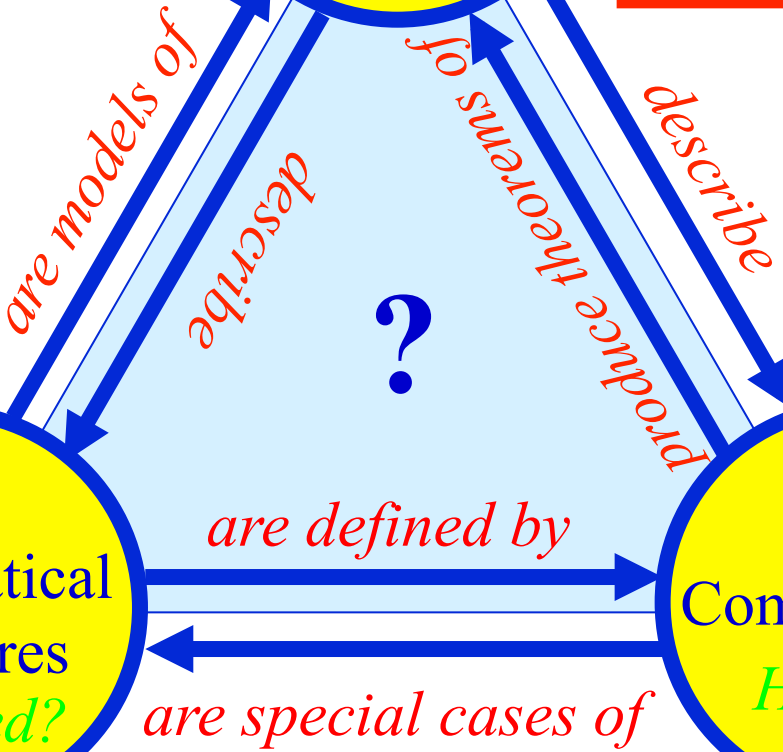
a  $\boxed{A}$  a  $\boxed{W}$ 
a  $\boxed{S}$  a  $\boxed{W}$  a  $\sim$   $\boxed{W}$ 
a  $\boxed{S}$  b  $\boxed{S}$  a  $\boxed{W}$  b  $\boxed{W}$  a b  $\bigvee$   $\boxed{W}$ 
a  $\boxed{S}$  b  $\boxed{A}$  c  $\boxed{S}$  a b c  $\boxed{T}$  d  $\boxed{W}$  a d c  $\boxed{T}$ 
a  $\boxed{T}$  b  $\boxed{S}$  a b  $\Rightarrow$   $\boxed{T}$  b  $\boxed{T}$ 
a  $\boxed{A}$  a a  $\bigvee$  a  $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  a a b  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  a b  $\bigvee$  b a  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  c  $\boxed{A}$  a b  $\Rightarrow$  c a  $\bigvee$  c b  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 

```

Formal systems
Decidable?

Mathematical structures
Defined?

Computations
Halting?



Do we live in a simulation? (The Matrix, Tipler, Bostrom, Schmidhuber, Lloyd...)



- McCabe: no, at least not a digital one
- Involves measure problem
- Unjustified to identify the 1D computational sequence with our 1D time
- Little said about how to identify the GR quantum universe with an evolving bit string.
- The “computer” doesn’t need to *evolve* the universe, merely *specify* it.




```

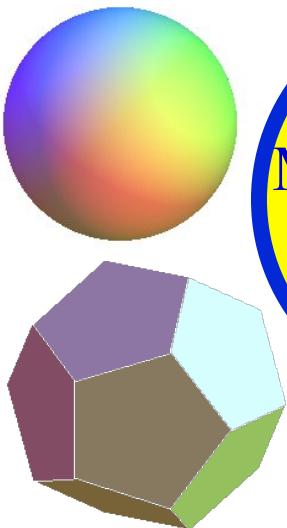
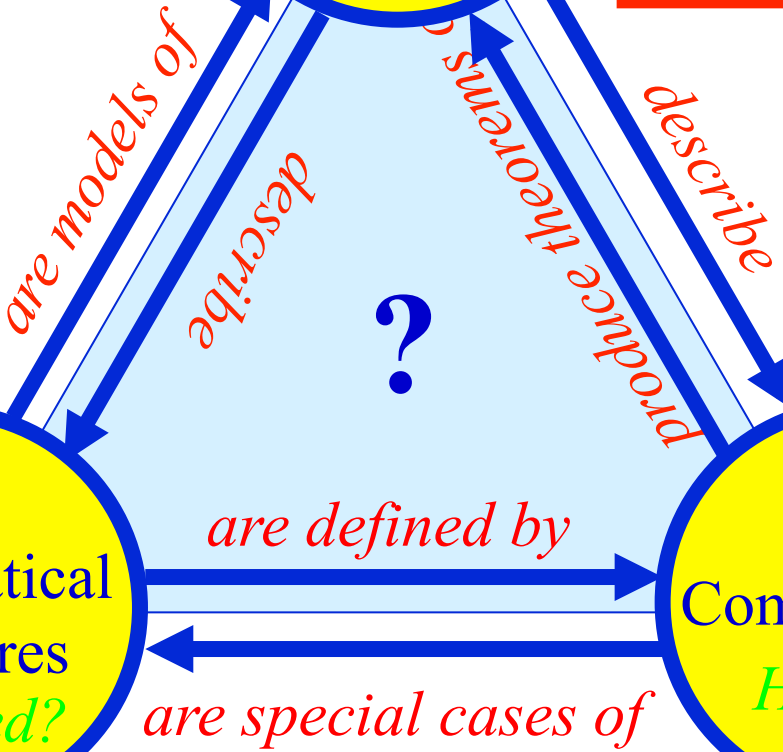
a  $\boxed{A}$  a  $\boxed{W}$ 
a  $\boxed{S}$  a  $\boxed{W}$  a  $\sim$   $\boxed{W}$ 
a  $\boxed{S}$  b  $\boxed{S}$  a  $\boxed{W}$  b  $\boxed{W}$  a b  $\bigvee$   $\boxed{W}$ 
a  $\boxed{S}$  b  $\boxed{A}$  c  $\boxed{S}$  a b c  $\boxed{T}$  d  $\boxed{W}$  a d c  $\boxed{T}$ 
a  $\boxed{T}$  b  $\boxed{S}$  a b  $\Rightarrow$   $\boxed{T}$  b  $\boxed{T}$ 
a  $\boxed{A}$  a a  $\bigvee$  a  $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  a a b  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  a b  $\bigvee$  b a  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 
a  $\boxed{A}$  b  $\boxed{A}$  c  $\boxed{A}$  a b  $\Rightarrow$  c a  $\bigvee$  c b  $\bigvee$   $\Rightarrow$   $\boxed{T}$ 

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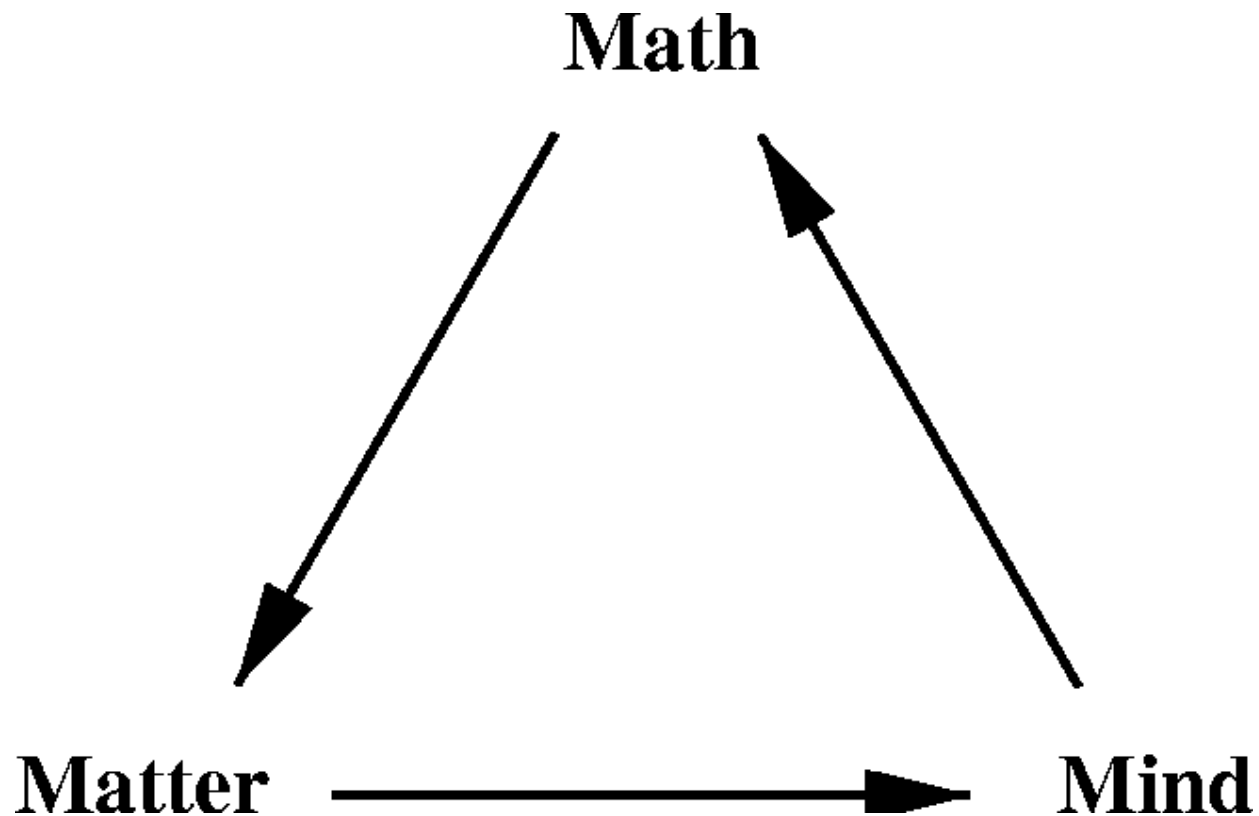
Formal systems
Decidable?

Mathematical structures
Defined?

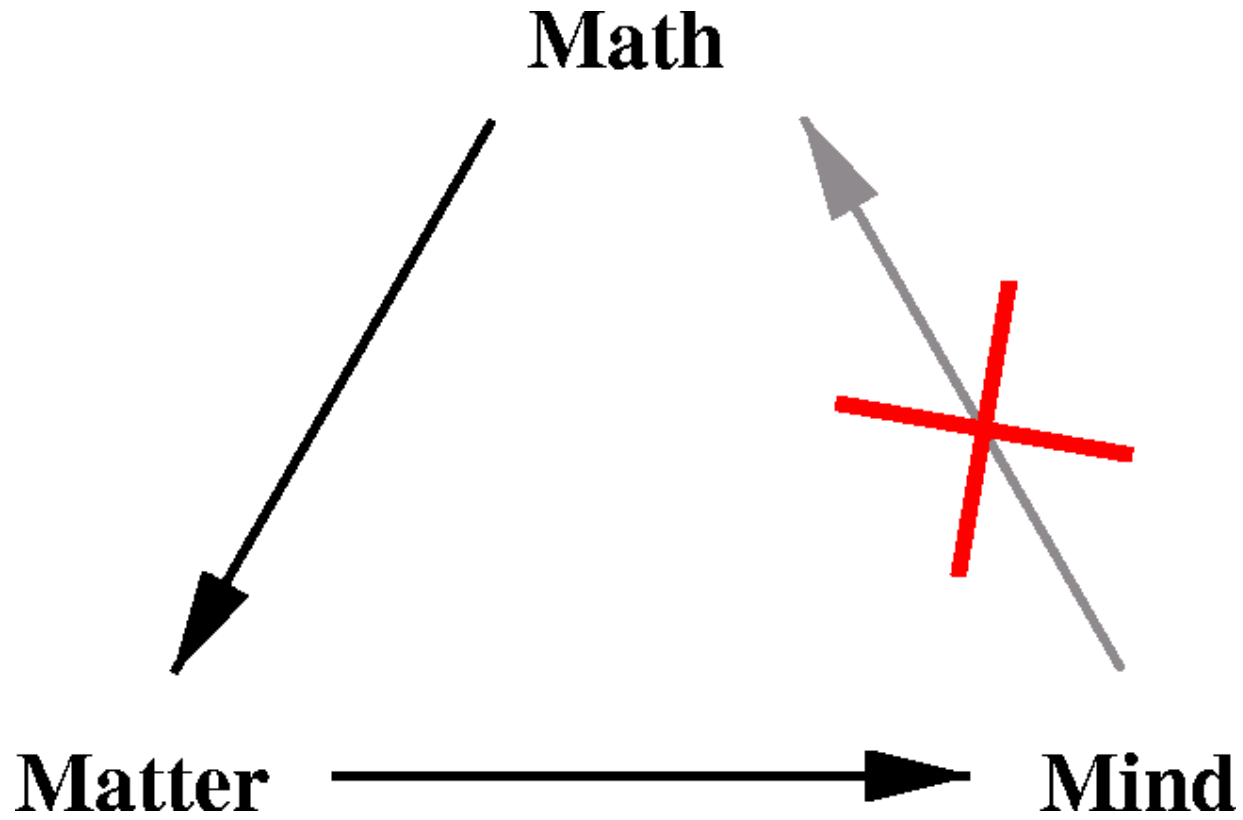
Computations
Halting?



Penrose's Bermuda triangle:



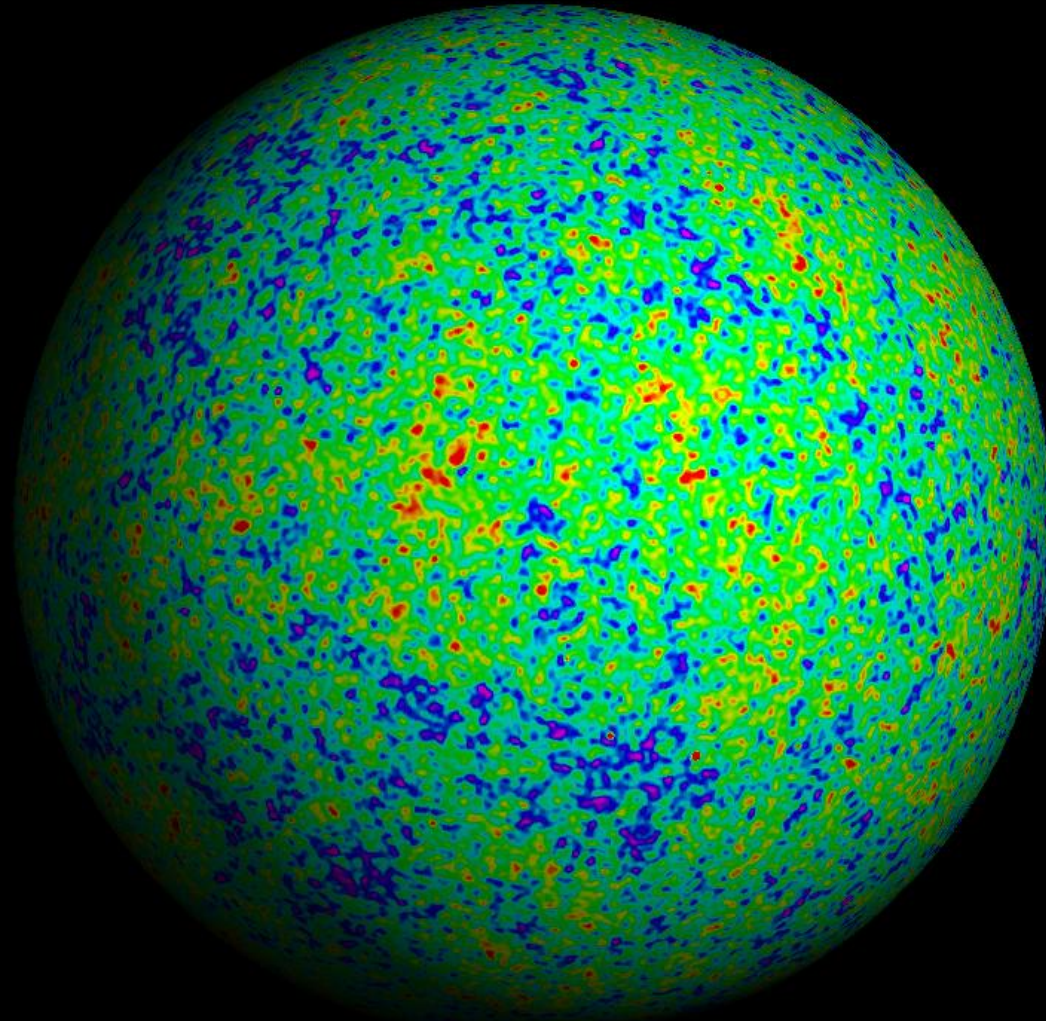
My guess:



Level IV multiverse

Q: What's the entropy of our universe?

A: About 10^{89} bits

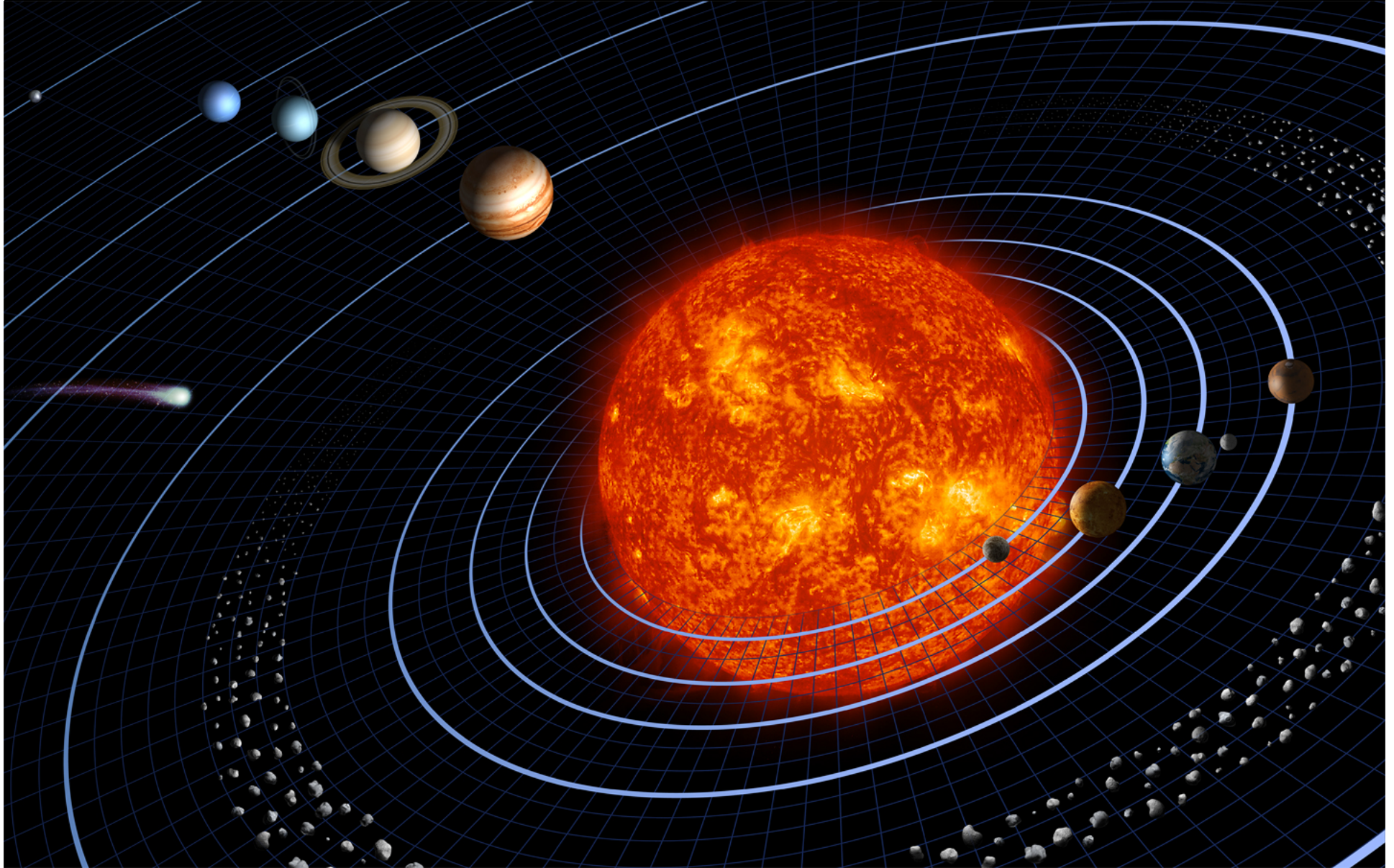


What does all that information really tell us?



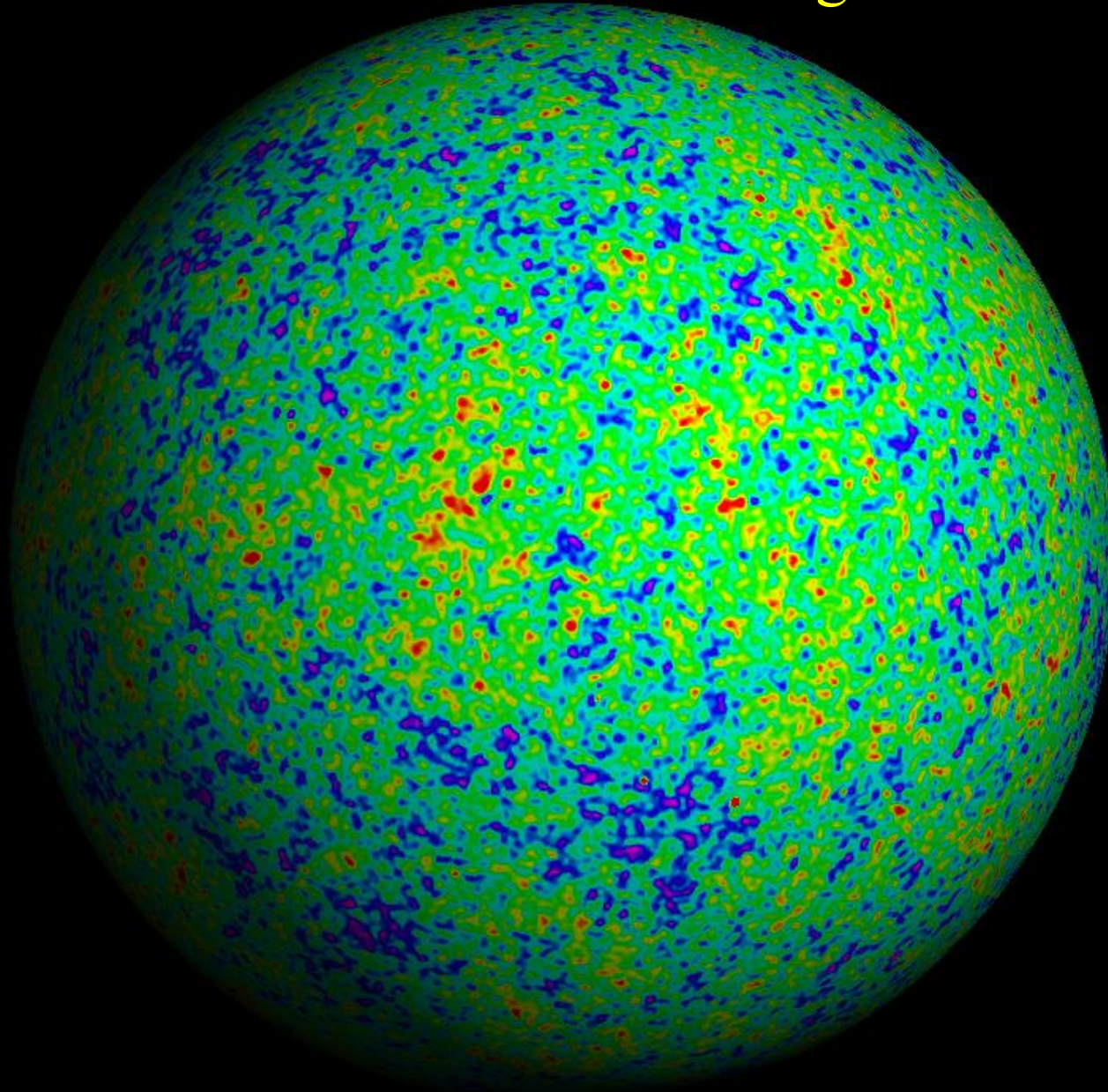
10^3 bits

How much of this information needs to go on the T-shirt?



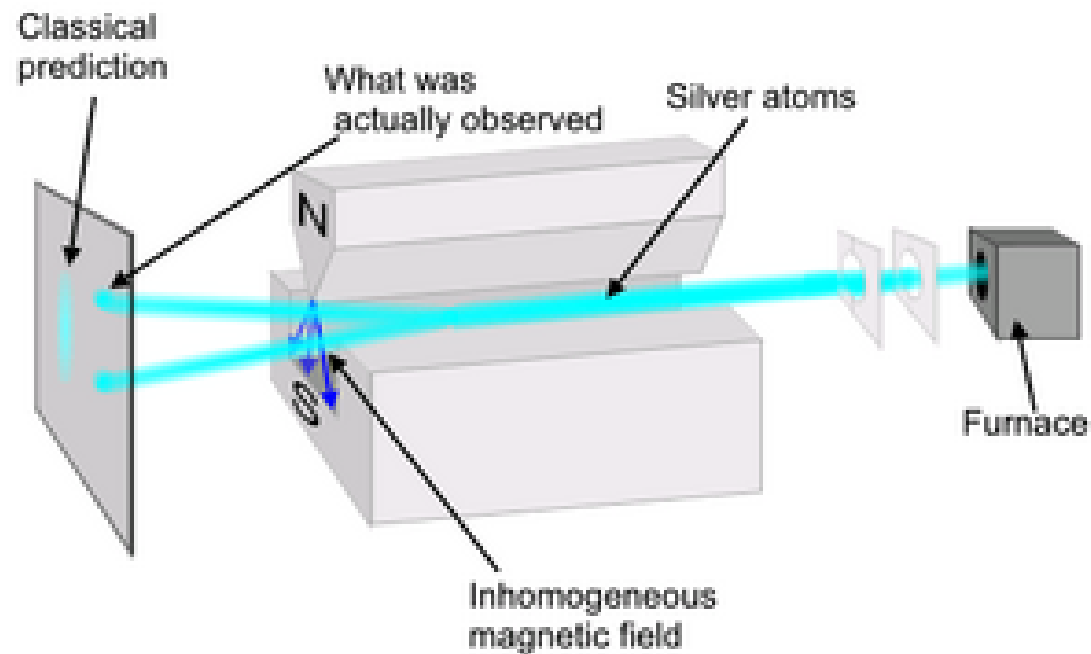
Very little

How much of this information needs to go on the T-shirt?

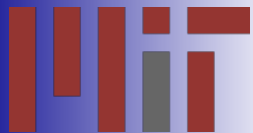


Very little

Quantum random number generator based on Stern-Gerlach apparatus:



Generic outcome: 101100100011001001110...
(Just our address in Hilbert space - not specified on T-shirt)



So what *does* go on the T-shirt?

Standard model parameters

Particle physics

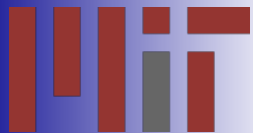
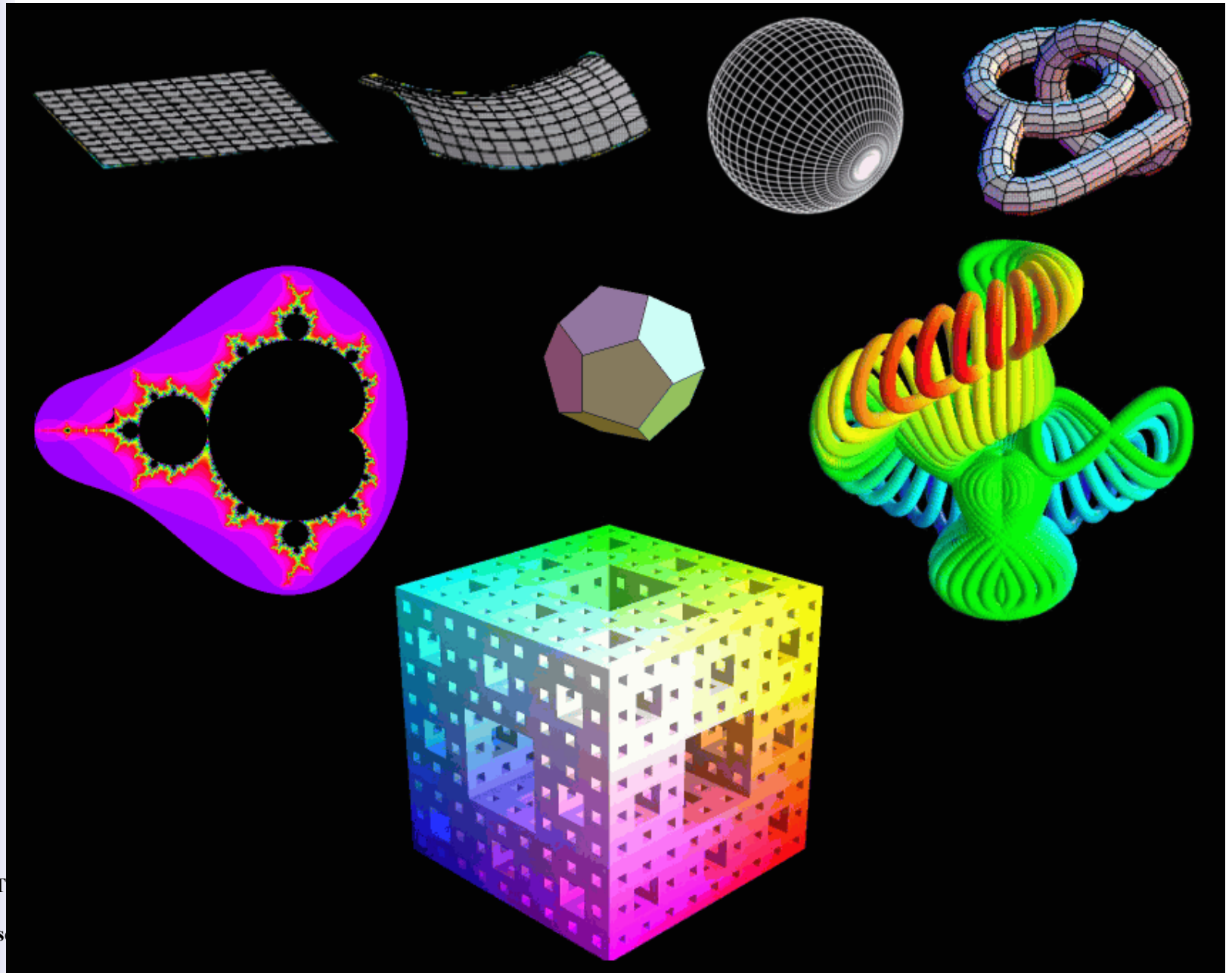
Cosmology

Parameter	Meaning	Measured value
g	Weak coupling constant at m_Z	0.6520 ± 0.0001
θ_W	Weinberg angle	0.48290 ± 0.00005
g_s	Strong coupling constant at m_Z	1.221 ± 0.022
μ^2	Quadratic Higgs coefficient	$\sim -10^{-33}$
λ	Quartic Higgs coefficient	$\sim 1?$
G_e	Electron Yukawa coupling	2.94×10^{-6}
G_μ	Muon Yukawa coupling	0.000607
G_τ	Tauon Yukawa coupling	0.0102156233
G_u	Up quark Yukawa coupling	0.000016 ± 0.000007
G_d	Down quark Yukawa coupling	0.00003 ± 0.00002
G_c	Charm quark Yukawa coupling	0.0072 ± 0.0006
G_s	Strange quark Yukawa coupling	0.0006 ± 0.0002
G_t	Top quark Yukawa coupling	1.002 ± 0.029
G_b	Bottom quark Yukawa coupling	0.026 ± 0.003
$\sin \theta_{12}$	Quark CKM matrix angle	0.2243 ± 0.0016
$\sin \theta_{23}$	Quark CKM matrix angle	0.0413 ± 0.0015
$\sin \theta_{13}$	Quark CKM matrix angle	0.0037 ± 0.0005
δ_{13}	Quark CKM matrix phase	1.05 ± 0.24
θ_{qcd}	CP-violating QCD vacuum phase	$< 10^{-9}$
G_{ν_e}	Electron neutrino Yukawa coupling	$< 1.7 \times 10^{-11}$
G_{ν_μ}	Muon neutrino Yukawa coupling	$< 1.1 \times 10^{-6}$
G_{ν_τ}	Tau neutrino Yukawa coupling	< 0.10
$\sin \theta'_{12}$	Neutrino MNS matrix angle	0.55 ± 0.06
$\sin 2\theta'_{23}$	Neutrino MNS matrix angle	≥ 0.94
$\sin \theta'_{13}$	Neutrino MNS matrix angle	≤ 0.22
δ'_{13}	Neutrino MNS matrix phase	?
ρ_Λ	Dark energy density	$(1.25 \pm 0.25) \times 10^{-123}$
ξ_b	Baryon mass per photon ρ_b/n_γ	$(0.50 \pm 0.03) \times 10^{-28}$
ξ_c	Cold dark matter mass per photon ρ_c/n_γ	$(2.5 \pm 0.2) \times 10^{-28}$
ξ_ν	Neutrino mass per photon $\rho_\nu/n_\gamma = \frac{3}{11} \sum m_{\nu_i}$	$< 0.9 \times 10^{-28}$
Q	Scalar fluctuation amplitude δ_H on horizon	$(2.0 \pm 0.2) \times 10^{-5}$
n_s	Scalar spectral index	0.98 ± 0.02

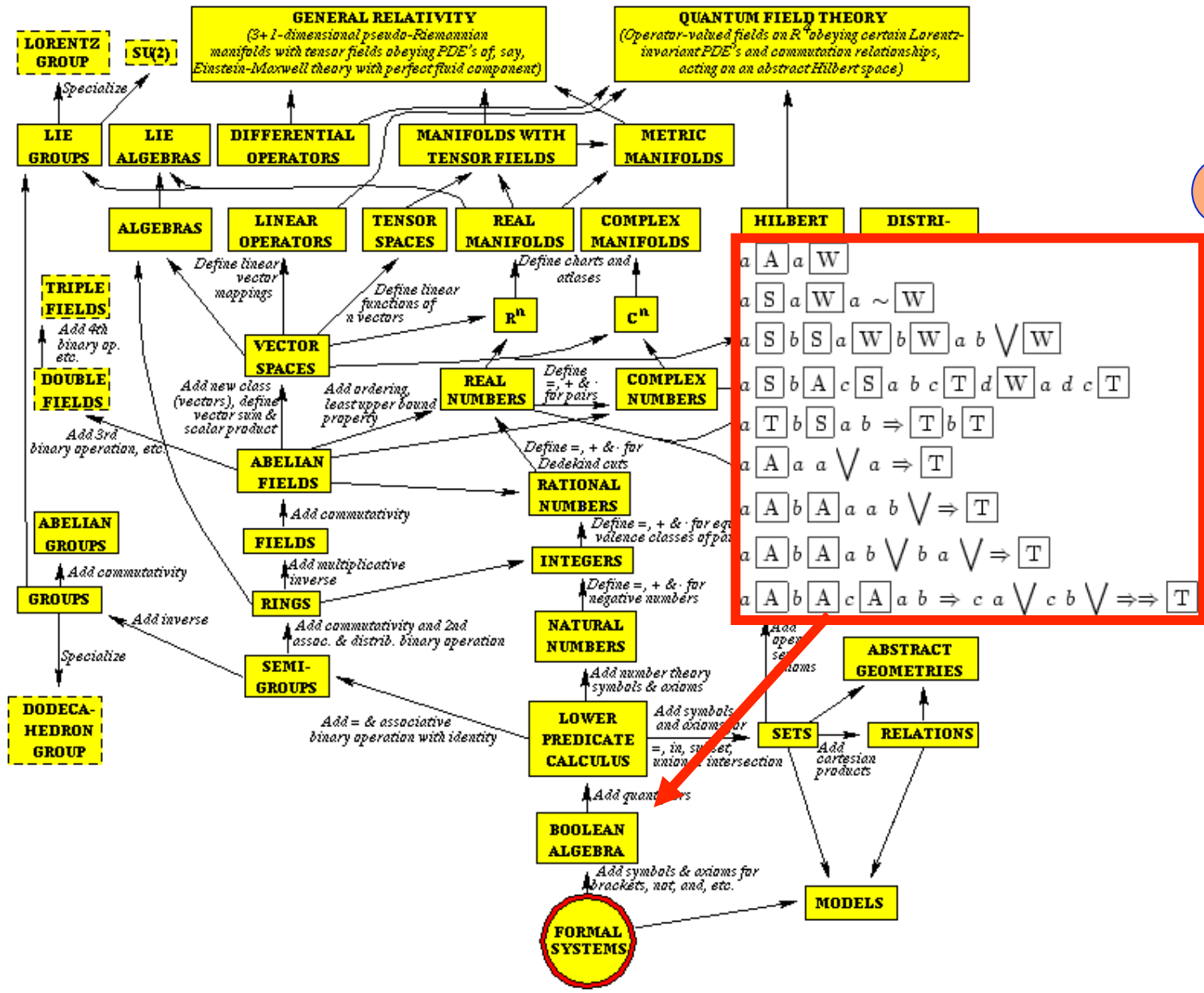
$$C = h = G = k_b = q_e = 1$$

MT,
Aguirre,
Rees &
Wilczek
2005

Multiverse level 4: other mathematical structures



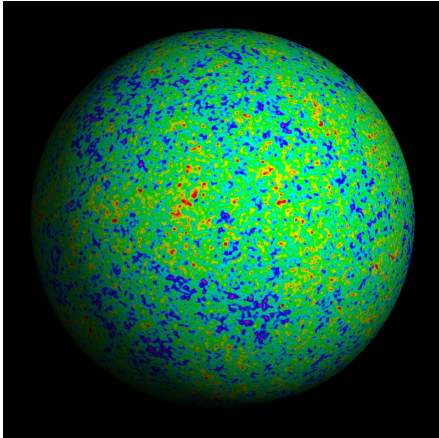
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May 13, 2011



If what we observe...

...requires more bits to describe than...

...a complete mathematical description of the world...



10^{89}
bits?

10^3
bits?



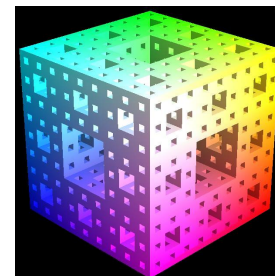
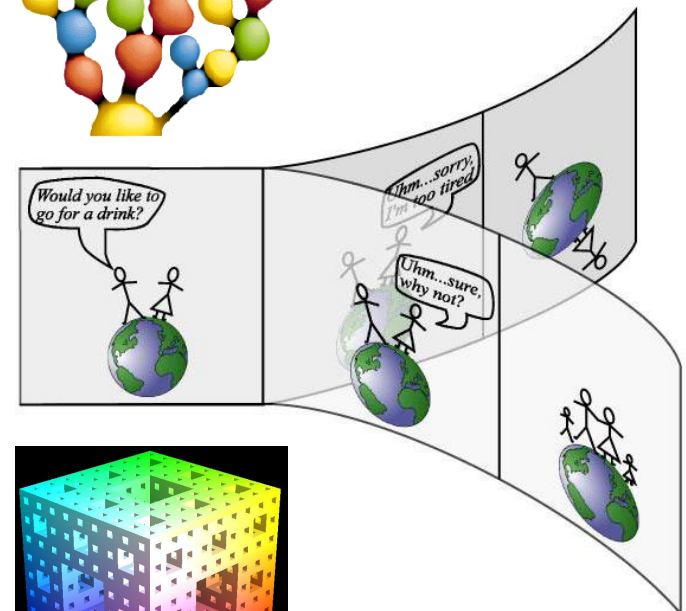
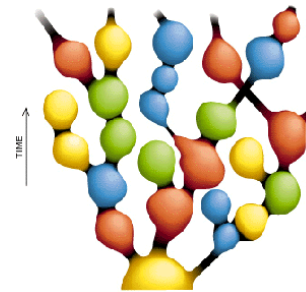
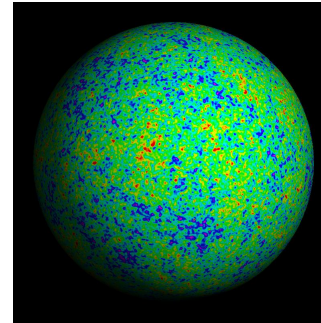
...then we're in a multiverse!



So if you're looking for a simple mathematical TOE, you're looking for a multiverse theory.

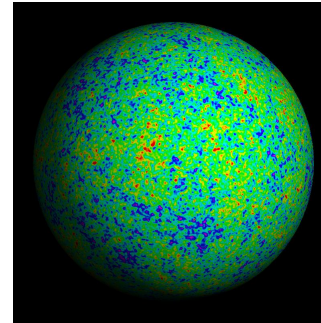
Which are the 4 multiverse levels?

- 1) Different Hubble volumes
- 2) Different post-inflationary regions
- 3) Different decohered branches of the quantum wavefunction
- 4) Different mathematical structures

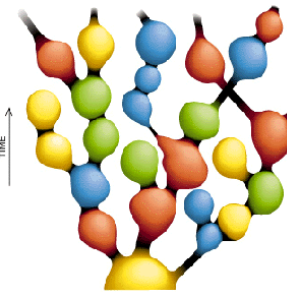


Where are the parallel universes?

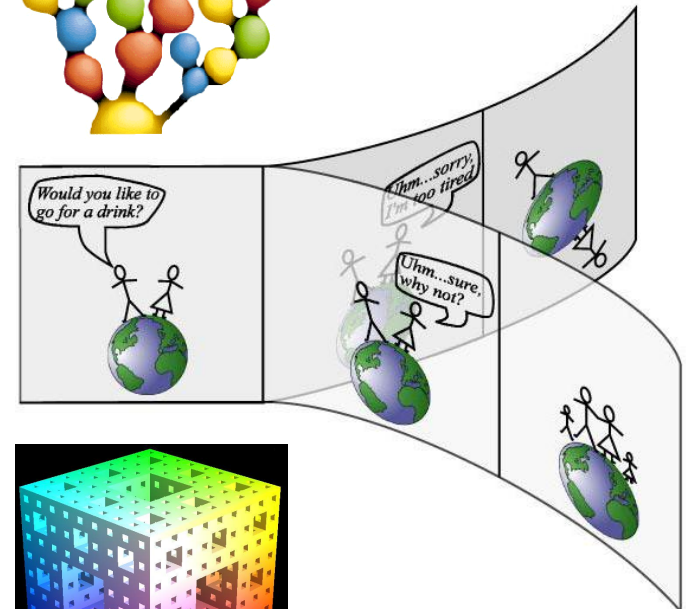
1) Far away in space



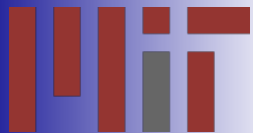
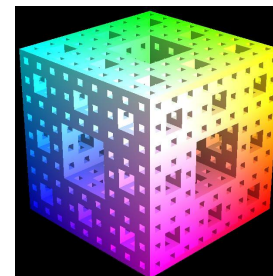
2) Infinitely far away in space



3) Elsewhere in Hilbert space

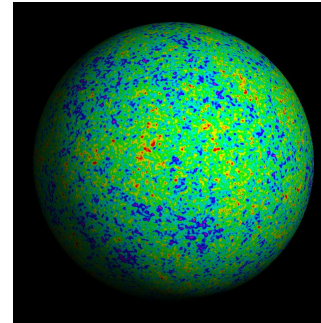


4) Elsewhere in “math space”

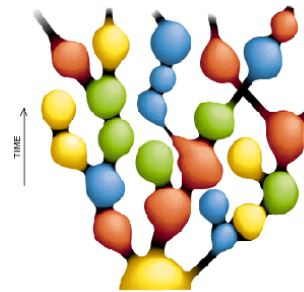


What are the 4 multiverse levels like?

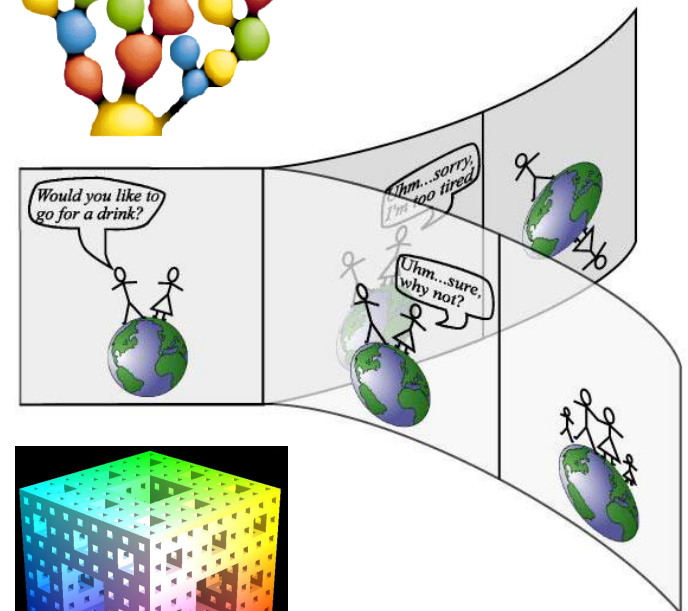
1) Same effective laws of physics, different initial conditions



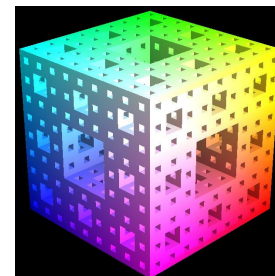
2) Same fundamental laws of physics, different effective laws (“bylaws”)



3) Nothing qualitatively new

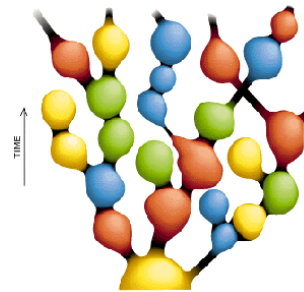
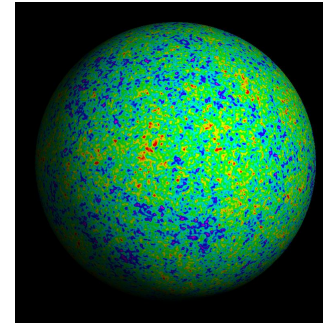


4) Different fundamental laws of physics

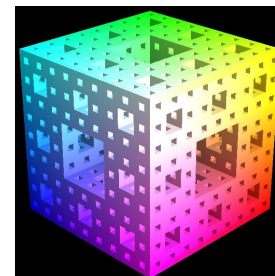
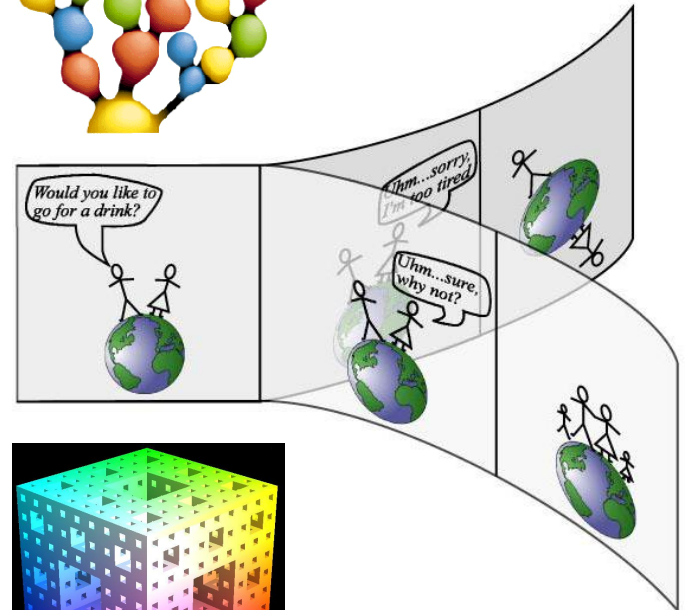
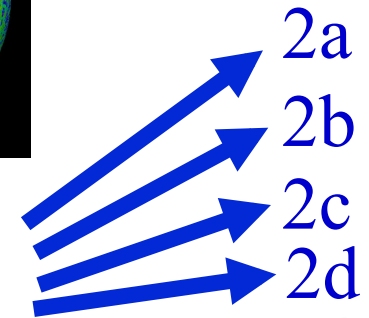


What are the 4 multiverse levels like?

- 1) Same effective laws of physics, different initial conditions
- 2) Same fundamental laws of physics, different effective laws
- 3) Nothing qualitatively new
- 4) Different fundamental laws of physics



↑
TIME

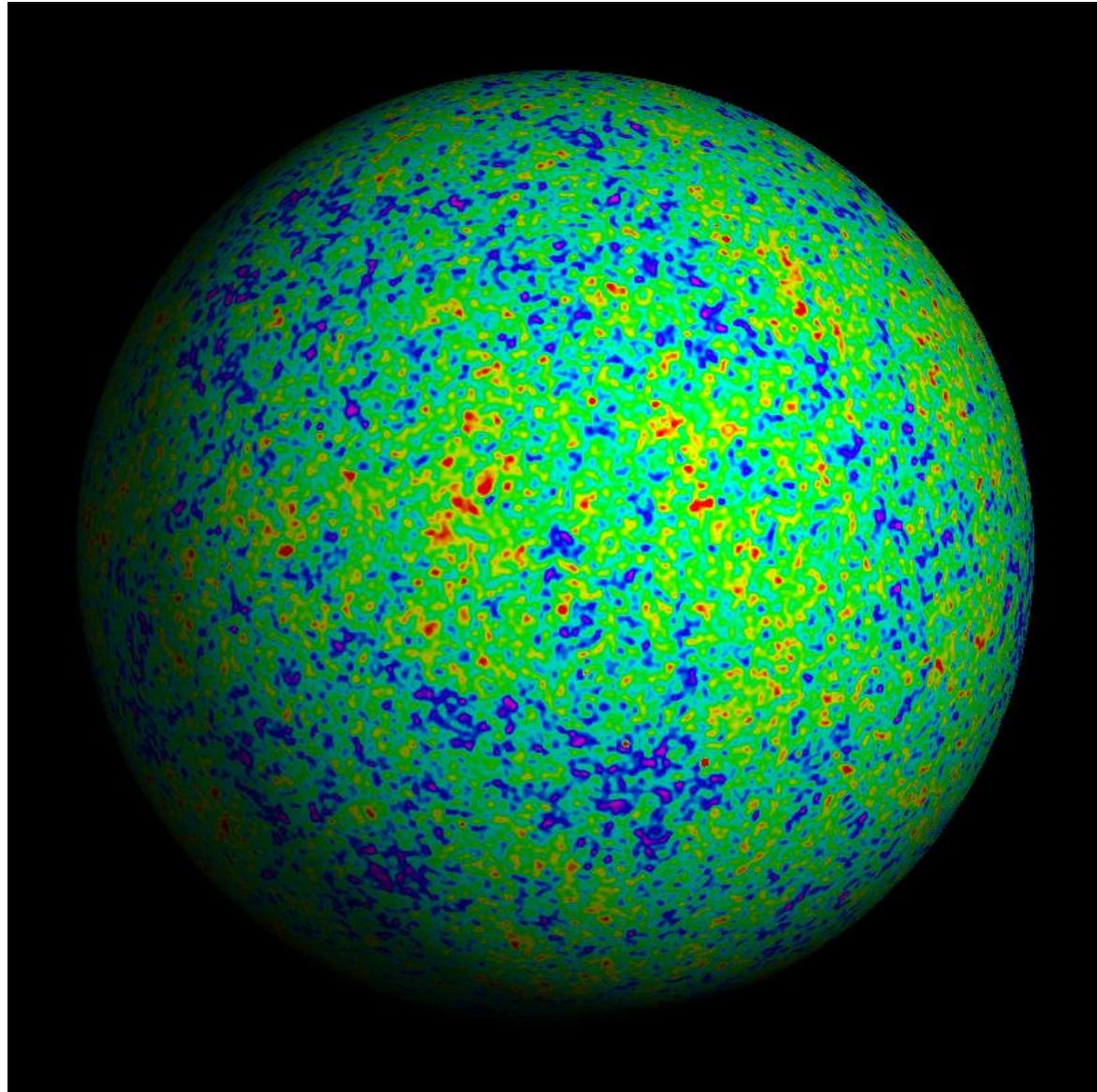


Evidence

How to test/falsify a multiverse theory:

Not necessary that rest of ensemble be observable

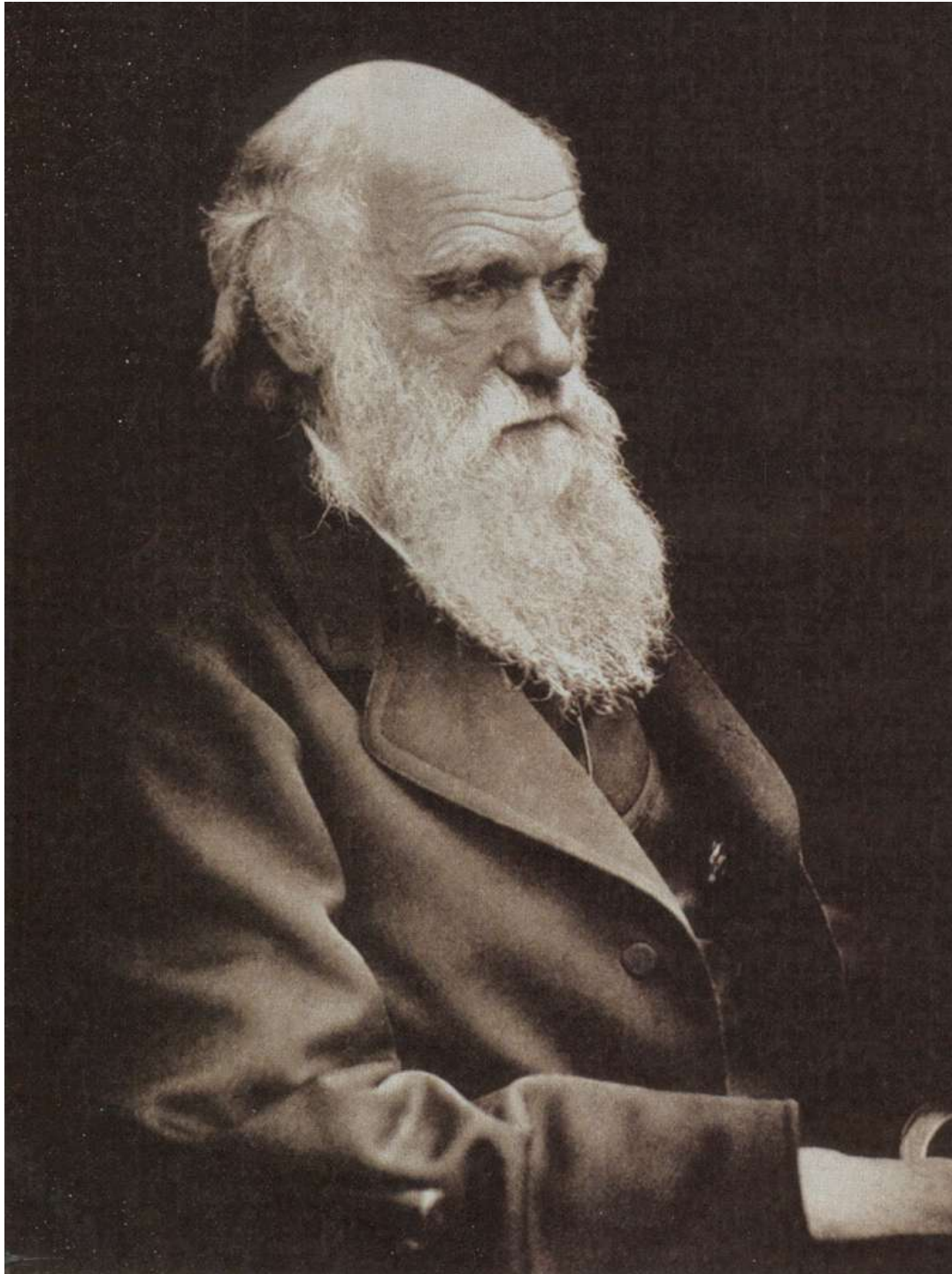
Example: the theory that there's no dark matter



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Bright Horizons Cruise
May 13, 2011

Sound too
crazy?

We're
not
taking
this guy
seriously
enough



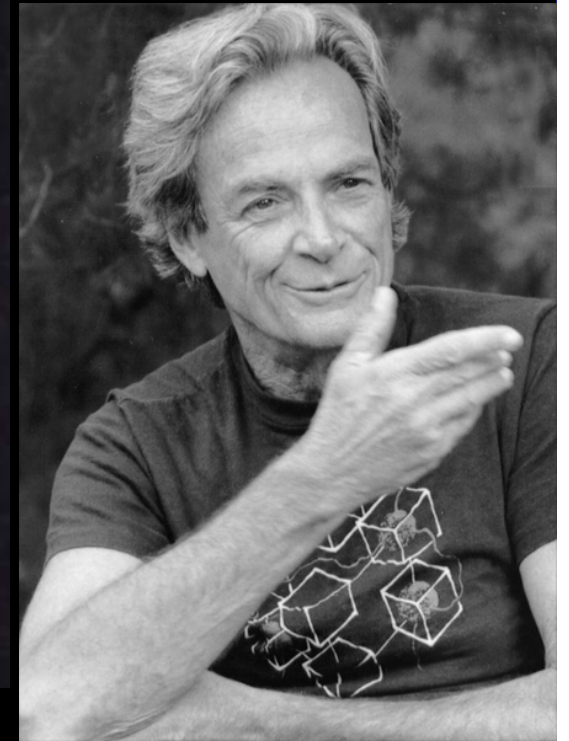
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The strongest form of the anthropic principle:



“The Universe must be such that we like it.”

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“The Universe must be such that we like it.”

